Kinematics of our Galaxy from the PMA and TGAS catalogues

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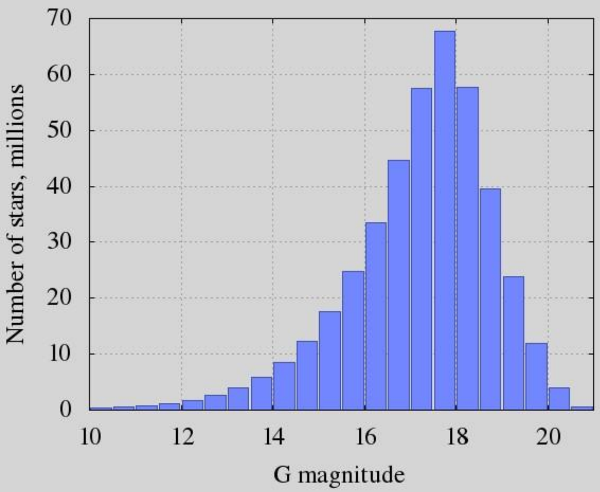
The used catalogues

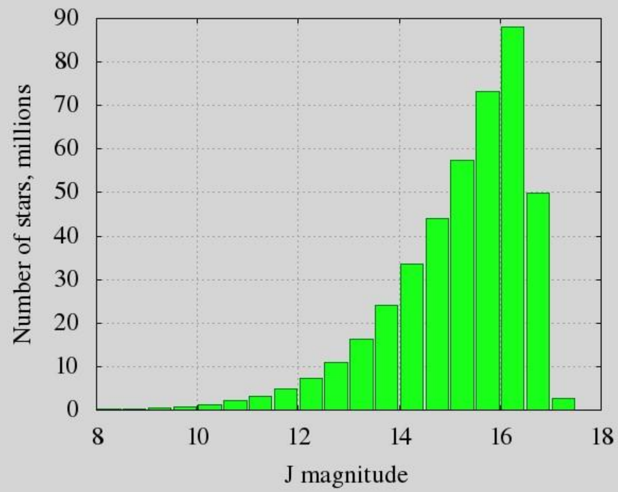
The **PMA** catalogue: combination of the *Gaia DR1* and *2MASS* catalogues
(**P**roper **M**otions **A**bsolute)

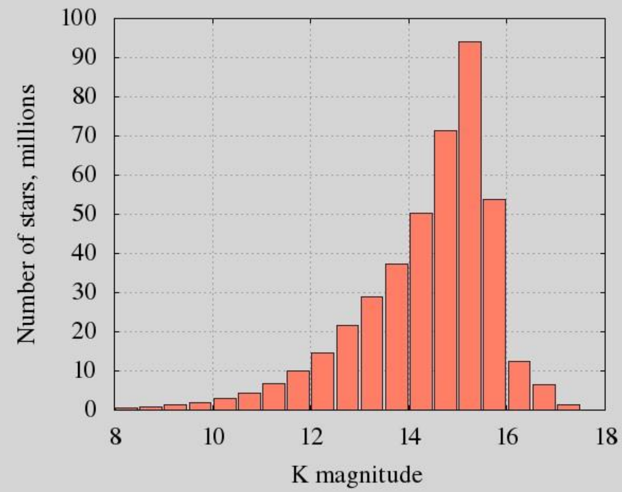
The **main purpose** is deriving **absolute proper motions**

- 421,446,141 objects
- 6 – 17.5 J and Ks magnitudes (*2MASS*)
- 8 – 21 G magnitudes (*Gaia DR1*)
- Gaia DR1 positions for the J2015 epoch

- $\sigma_{\alpha,\delta} \approx$ 10 mas (taken from Gaia DR1)
- $\sigma_{\mu\alpha,\delta} \approx$ from 2-5 mas/yr for 10 < G < 17 to 5-10 mas/yr for fainter G magnitudes



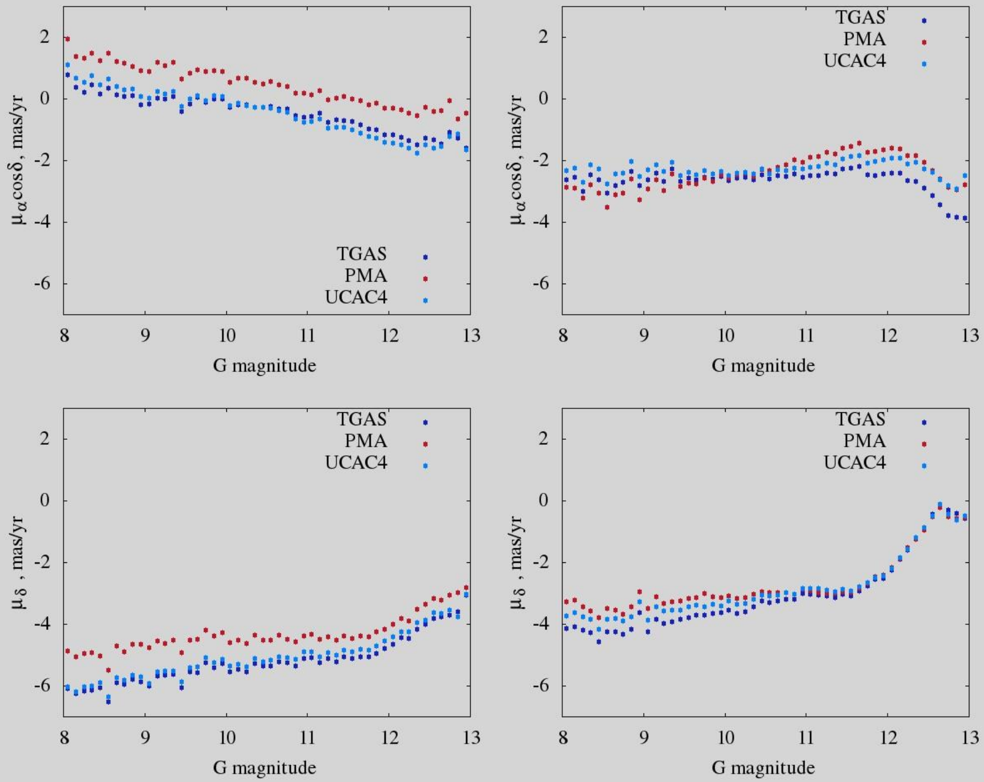




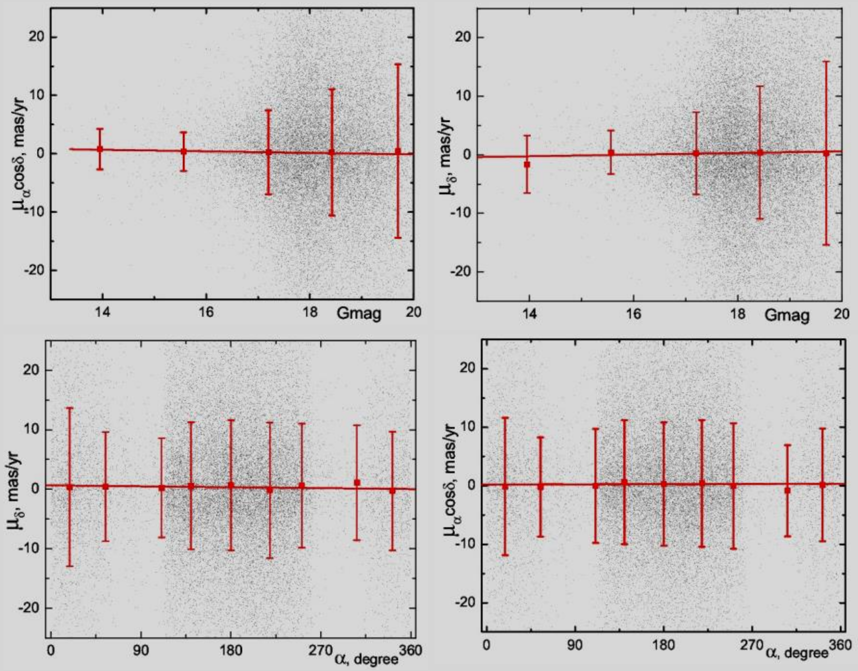
Comparison of the **PMA** with the **TGAS** and **UCAC4**

The Northern hemisphere
+12° < δ < 90°

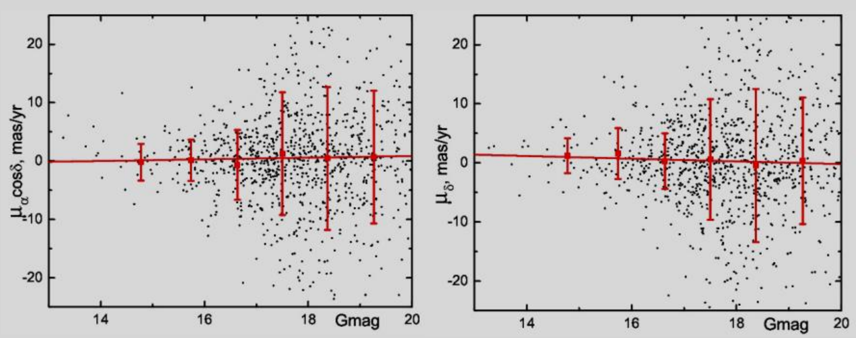
The Southern hemisphere
-90° < δ < +12°



Scatter of individual proper motions for LQAC3 quasars



Scatter of individual proper motions for ICRF quasars



The system of PMA proper motions is **independent** on ICRF and HCRF, and together with its own positions in the range from 14 to 21 magnitude represents an **independent realization** of a quasi-inertial reference frame in the optical and near infrared wavelength ranges.

(Akhmetov, Fedorov, Velichko, Shulga)

MNRAS, 2017
10.1093/mnras/stx812

Stellar velocity field

$$\mathbf{U}(l,b) = V_r/r\mathbf{e}_r + \kappa\mu_l cos\, b\mathbf{e}_l + \kappa\mu_b\mathbf{e}_b$$

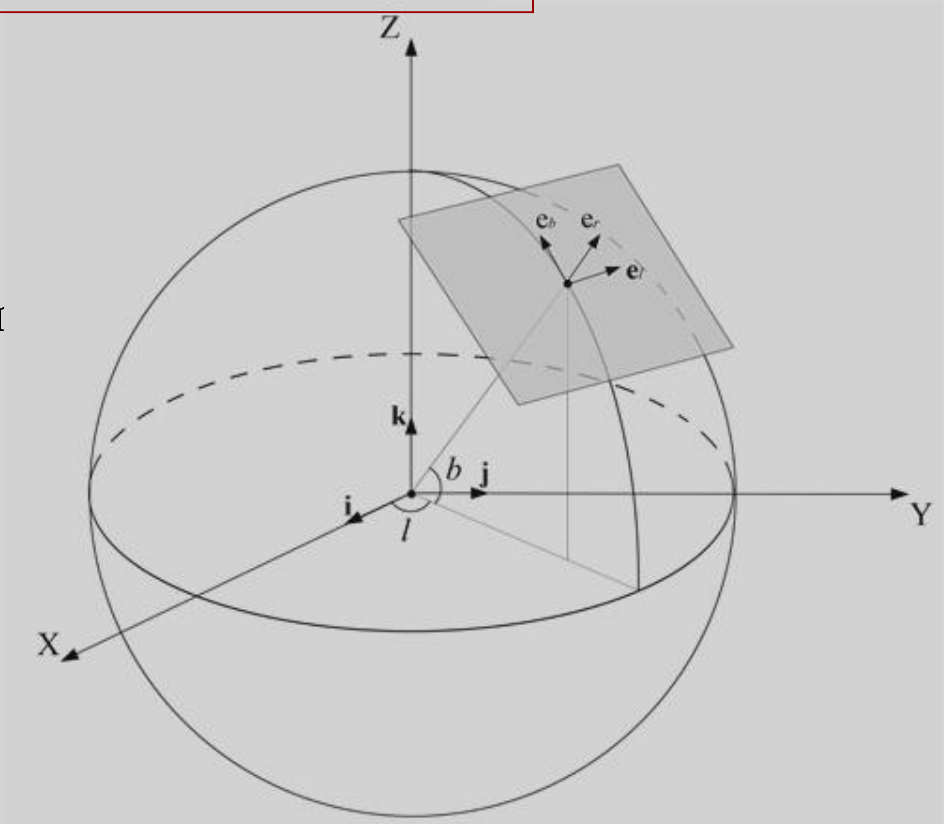
We have only proper motions, so

$$\mathbf{U}(l,b) = \kappa\mu_l cos\, b\mathbf{e}_l + \kappa\mu_b\mathbf{e}_b$$

V_r - radial velocity

$\mu_l,\ \mu_b$ - proper motion components

$\kappa = 4.738$ - converting factor from mas/yr to km/s kpc^{-1}



Two approaches for deriving kinematic parameters of the Galaxy

1. Physical approach: Physical model + LSM = kinematic parameters

e.g.: **Ogorodnikov – Milne** model

$$\mathbf{U}(l,b) = \mathbf{V}_0 + \Omega \times \mathbf{r} + \mathbf{M}^+ \times \mathbf{r}$$

11 parameters:

$\mathbf{V}_0 : U, V, W$

(Solar motion)

$\Omega : \omega_1, \omega_2, \omega_3$

(Rigid-body rotation)

$\mathbf{M}^+ : M_{11}^+ - M_{22}^+, \quad M_{33}^+ - M_{22}^+$

(Contraction - expansion)

$M_{12}^+, \ M_{13}^+, \ M_{23}^+$

(Deformation in planes)

(Ogorodnikov, 1965)

2. Mathematical (formal) approach : decomposition of data onto the system of ortonormal functions on the sphere

Vector Spherical Harmonics

$$\mathbf{U}(l,b) = \sum_{nkp} t_{nkp}\mathbf{T}_{nkp} + \sum_{nkp} s_{nkp}\mathbf{S}_{nkp}$$

Thoroidal:

$$\mathbf{T}_{nkp} = T^l_{nkp}\mathbf{e}_l + T^b_{nkp}\mathbf{e}_b = \frac{1}{\sqrt{n(n+1)}} \times \left(\frac{\partial K_{nkp}(l,b)}{\partial b}\mathbf{e}_l - \frac{1}{\cos b}\frac{\partial K_{nkp}(l,b)}{\partial l}\mathbf{e}_b \right)$$

Spheroidal:

$$\mathbf{S}_{nkp} = S^l_{nkp}\mathbf{e}_l + S^b_{nkp}\mathbf{e}_b = \frac{1}{\sqrt{n(n+1)}} \times \left(\frac{1}{\cos b}\frac{\partial K_{nkp}(l,b)}{\partial l}\mathbf{e}_l + \frac{\partial K_{nkp}(l,b)}{\partial l}\mathbf{e}_b \right)$$

where: $$K_{nkp}(l,b) = R_{nk} \times \begin{cases} P_{n,0}(b), & k=0, p=1, \\ P_{nk}(b)\sin kl, & k\neq 0, p=0, \\ P_{nk}(b)\cos kl, & k\neq 0, p=1 \end{cases}$$

$$R_{nk} = \sqrt{\frac{2n+1}{4\pi}} \begin{cases} \sqrt{\frac{2(n-k)!}{(n+k)!}}, & k>0, \\ 1, & k=0 \end{cases}$$

(**Vityazev et al.**, 2009)

(**Mignard & Klioner**, 2012)

Stellar velocity field
in the tangential plane
(after projection onto the Galactic coordinate system)

via **O-M model**:

$$\begin{cases} \kappa\,\mu_l\,cos\,b = U/r\,sin\,l - V/r\,cos\,l - \omega_1\,sin\,b\,cos\,l - \omega_2\,sin\,b\,sin\,l + \omega_3\,cos\,b - M_{13}^{+}\,sin\,b\,sin\,l + \\ \quad M_{23}^{+}\,sin\,b\,cos\,l + M_{12}^{+}\,cos\,b\,cos\,2l - \frac{1}{2}M_{11}^{+}\,cos\,b\,sin\,2l + \frac{1}{2}M_{22}^{+}\,cos\,b\,sin\,2l \\ \kappa\,\mu_b = U/r\,cos\,l\,cos\,b + V/r\,sin\,l\,sin\,b - W/r\,cos\,b + \omega_1\,sin\,l - \omega_2\,cos\,l - \frac{1}{2}M_{12}^{+}\,sin\,2b\,sin\,2l + \\ \quad M_{13}^{+}\,cos\,2b\,cos\,l + M_{23}^{+}\,cos\,2b\,sin\,l - \frac{1}{2}M_{11}^{+}\,sin\,2b\,cos^2 l - \frac{1}{2}M_{22}^{+}\,sin\,2b\,sin^2 l + \frac{1}{2}M_{33}^{+}\,sin\,2b \end{cases}$$

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via **VSH**:

First, we select all significant harmonics,

$$\begin{cases} \kappa\mu_l\,cos\,b = \sum_{nkp} t_{nkp}\mathbf{T}^{l}_{nkp}(l,b) + \sum_{nkp} s_{nkp}\mathbf{S}^{l}_{nkp}(l,b) \\ \kappa\mu_b = \sum_{nkp} t_{nkp}\mathbf{T}^{b}_{nkp}(l,b) + \sum_{nkp} s_{nkp}\mathbf{S}^{b}_{nkp}(l,b) \end{cases}$$

(**Vityazev & Tsvetkov**, 2015)

Kinematic parameters based on the Ogorodnikov-Milne model from the PMA catalogue data (common with the TGAS) depending on distance.

2,048,407 objects

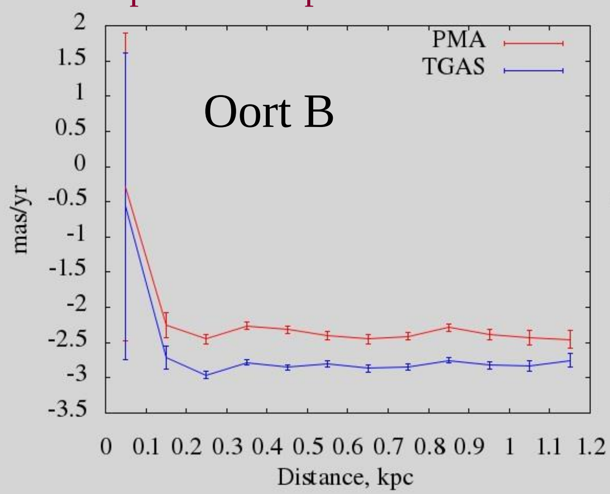
Mixed spectral composition

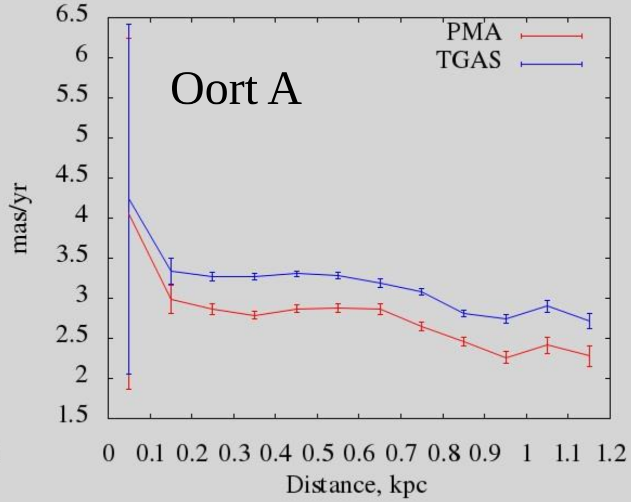
	Distance, pc	50	150	250	350	450	550	650	750	850	950	1050	1150
km/s	U	5,44	9,17	9,30	9,76	10,25	10,33	10,13	11,44	12,45	11,93	13,05	13,62
	V	10,73	17,13	16,84	16,54	16,17	16,34	17,11	18,93	20,77	23,52	25,93	29,11
	W	4,57	7,12	6,89	6,58	6,60	6,70	6,30	6,71	6,62	6,62	7,31	6,75
mas/yr	ω_3 (Oort B)	-0,29	-2,24	-2,44	-2,26	-2,31	-2,4	-2,44	-2,4	-2,28	-2,38	-2,42	-2,45
	ω_2	-1,49	-0,98	-0,39	-0,18	-0,16	-0,24	-0,29	0,01	0,05	0,15	-0,07	-0,02
	ω_1	-0,52	0,33	0,30	0,11	0,00	-0,04	0,12	0,05	0,07	0,12	0,01	0,06
	M_{12}^+ (Oort A)	4,05	2,99	2,87	2,79	2,87	2,88	2,86	2,65	2,46	2,26	2,42	2,28
	M_{13}^+	-1,42	-0,90	-0,51	-0,43	-0,48	-0,47	-0,45	-0,32	-0,21	-0,01	-0,33	-0,39
	M_{23}^+	0,20	-0,62	-0,31	-0,07	-0,02	0,04	0,02	-0,09	-0,09	-0,29	-0,13	-0,07
	$M_{11}^+ - M_{22}^+$	0,49	-1,50	-1,48	-2,10	-2,20	-1,90	-1,79	-1,58	-1,21	-1,59	-1,14	-1,13
	$M_{33}^+ - M_{22}^+$	-0,22	0,00	-0,16	-0,09	-9,27	0,58	3,04	2,91	2,99	0,50	0,82	0,22

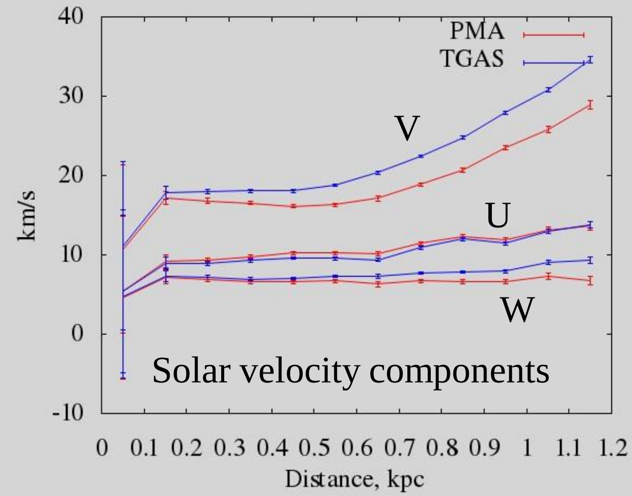
Distance = $1/\pi$

Kinematic parameters for PMA and TGAS catalogues via the O-M model

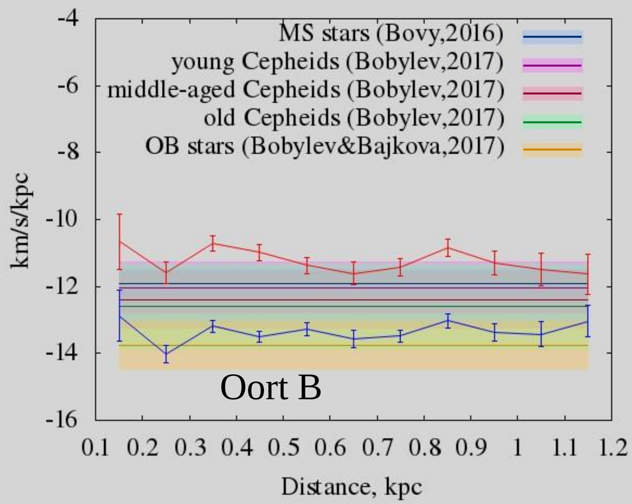
Mixed spectral composition

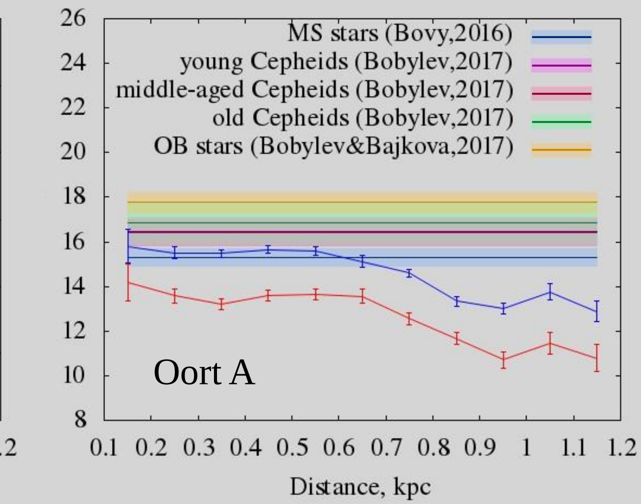


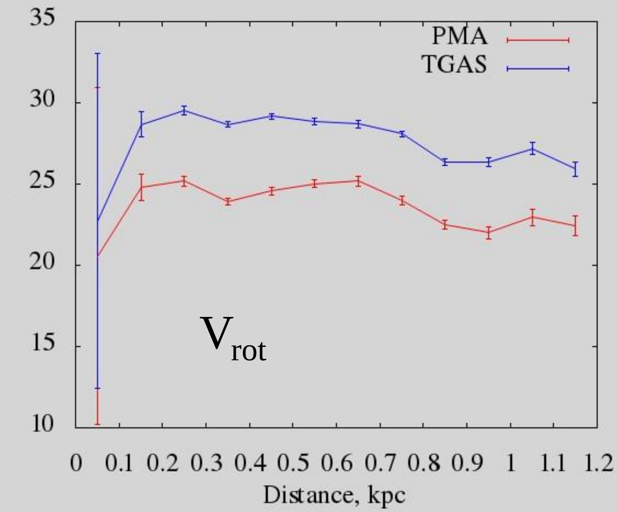




Comparison with values derived by other authors from Gaia DR1 data







$V_{rot} = (B – A) \cdot 4.738$ [km/s/kpc]

(Bovy,2016): [J.Bovy, Mon Not R Astron Soc Lett 468 (1): L63-L67] ***304,267*** *TGAS Main Sequence stars*

(Bobylev,2017): [V.V. Bobylev, Astron. Lett. 43: 152.] ***75*** *young Cepheids,* ***99*** *middle-aged Cepheids,* ***86*** *old Cepheids of Gaia DR1*

(Bobylev&Bajkova,2017): [V.V. Bobylev & A.T. Bajkova Astron. Lett. 43: 159] ***238*** *OB stars of Gaia DR1*

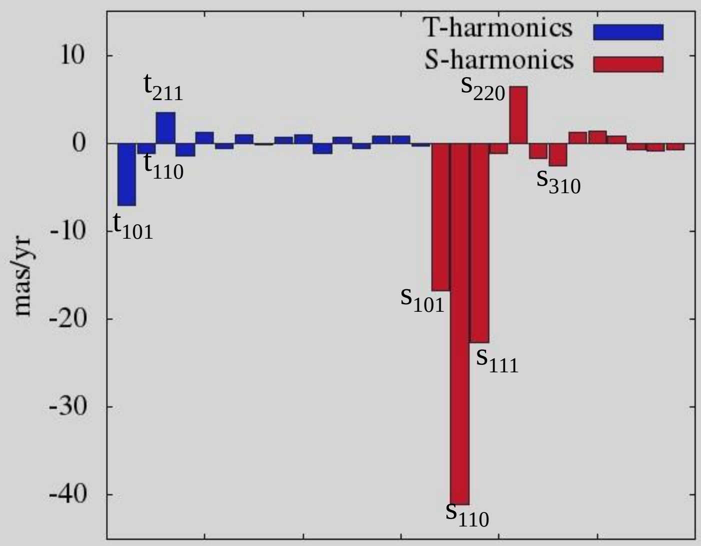
Distance = $1/\pi$

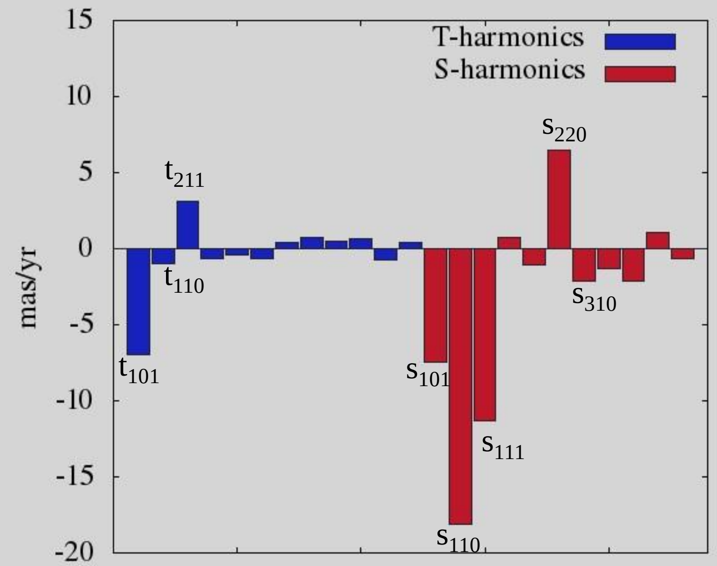
Kinematic spectra of the velocity field via VSH

t_{101} = Oort B
s_{220} = Oort A

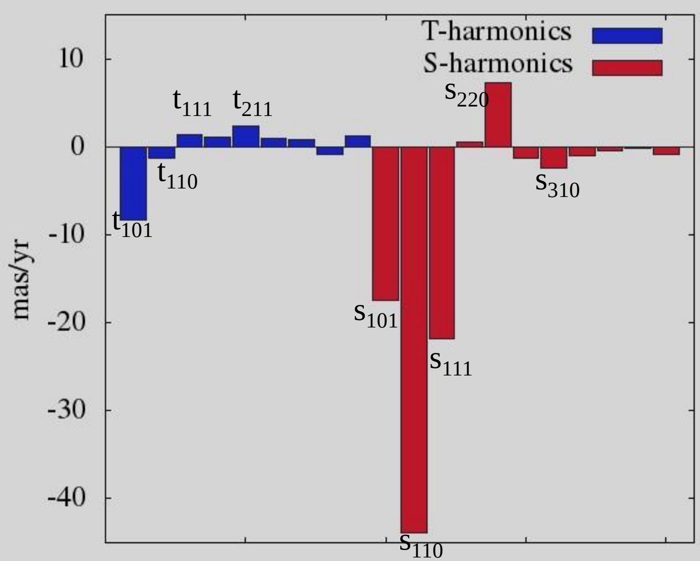
Distance to the sample: R = **250 pc** R = **550 pc**

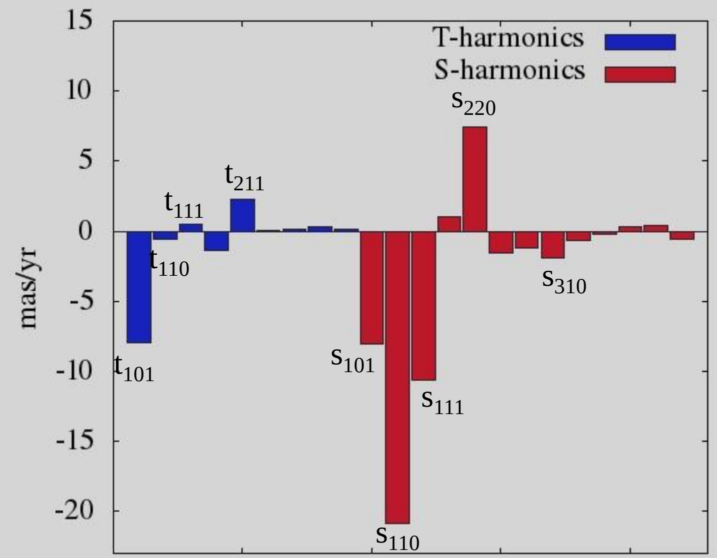
PMA





TGAS





Relations between VSH coefficients and O-M model parameters

Koefficient	Meaning
t_{111}	$2.89\omega_1$
t_{110}	$2.89\omega_2$
t_{101}	$2.89\omega_3$
s_{111}	$-2.89U/r$
s_{110}	$-2.89V/r$
s_{101}	$-2.89W/r$
s_{201}	$-0.65M^+_{11} - 0.65M^+_{22} + 1.29M^+_{33}$
s_{210}	$2.24M^+_{23}$
s_{211}	$2.24M^+_{13}$
s_{220}	$2.24M^+_{12}$
s_{211}	$1.12M^+_{11}$ - $1.12M^+_{22}$

(Vityazev & Tsvetkov, 2009)

Kinematic parameters for **PMA** and **TGAS** catalogues via VSH

PMA

	Distance, pc	150	250	350	450	550	650	750	850	950	1050	1150
km/s	U	9,17	9,30	9,76	10,25	10,33	10,13	10,92	11,27	10,02	9,66	9,06
	V	17,13	16,84	16,54	16,17	16,34	17,11	19,08	18,64	18,45	18,68	17,22
	W	7,12	6,89	6,58	6,60	6,70	6,30	6,01	5,16	3,31	1,24	0,13
mas/yr	ω_3 (Oort B)	-2,15	-2,44	-2,27	-2,32	-2,40	-2,35	-2,41	-2,29	-2,38	-2,43	-2,47
	ω_2	-1,02	-0,39	-	-	-	-	-	-	-	-	-
	ω_1	-	0,31	-	-	-	-	-	-	-	-	-
	M_{12}^+ (Oort A)	3,05	2,85	2,78	2,86	2,88	2,70	2,65	2,46	2,26	2,39	2,27
	M_{13}^+	-0,84	-0,52	-0,43	-0,47	-0,47	10,78	7,94	-	-	-	-
	M_{32}^+	-0,65	-	-	-	-	-	-	-	-	-	-
	$M_{11}^+ - M_{22}^+$	-1,50	-1,50	-2,08	-2,17	-1,91	-1,67	-1,58	-1,21	-1,59	-1,13	-1,12
	$M_{33}^+ - M_{22}^+$	0,10	-	-	-0,60	-0,40	-	-	0,02	-	-	-
	RMS	0,76	0,34	0,27	0,28	0,30	0,23	0,30	0,28	0,38	0,47	0,57

Extra-model:

s_{310} t_{211}

TGAS

	Distance, pc	150	250	350	450	550	650	750	850	950	1050	1150
km/s	U	8,84	8,59	8,87	8,98	9,44	9,38	10,90	11,99	11,46	13,00	13,80
	V	18,04	18,29	18,25	18,01	18,49	19,86	22,44	24,79	27,97	30,81	34,60
	W	7,25	6,96	6,84	6,84	7,02	7,13	7,72	7,81	7,95	9,02	9,35
mas/yr	ω_3 (Oort B)	-2,57	-2,88	-2,70	-2,78	-2,76	-2,77	-2,84	-2,76	-2,82	-2,84	-2,77
	ω_2	-0,94	-0,44	-0,23	-	-	-	-	-	-	-	-
	ω_1	0,00	0,46	0,35	-	-	-	-	-	-	-	-
	M_{12}^+ (Oort A)	3,36	3,28	3,30	3,37	3,34	3,13	3,08	2,82	2,74	2,90	2,71
	M_{13}^+	-0,69	-	-	-	-	-	-	-	-	-	-
	M_{32}^+	-	-	-	-	-	-	-	-	-	-	-
	$M_{11}^+ - M_{22}^+$	-1,19	-1,17	-1,50	-1,75	-1,42	-1,35	-1,31	-0,84	-1,35	-	-
	$M_{33}^+ - M_{22}^+$	0,23	-0,14	-0,09	-0,01	0,10	0,03	-0,07	0,41	0,09	0,56	-
	RMS	0,63	0,24	0,17	0,15	0,15	0,18	0,22	0,23	0,28	0,38	0,45

Reduced proper motions (RPM) and selection of **red dwarfs**

Absolute magnitude

$M_{K_S} = K_S + 5 + 5\lg(\pi)$

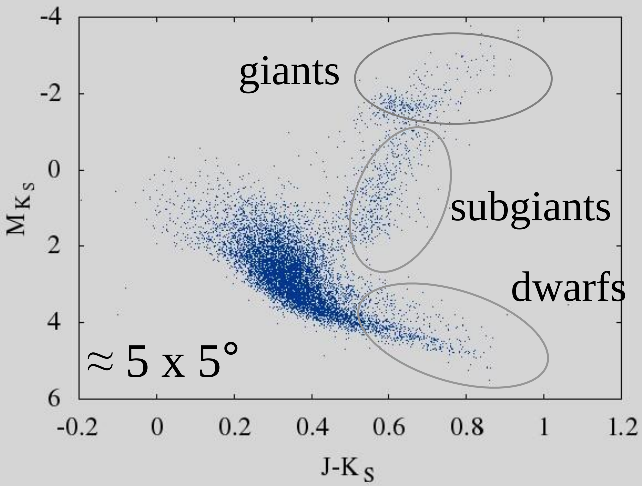
$M_{K_S} = a \cdot RPM(K_S) + b$

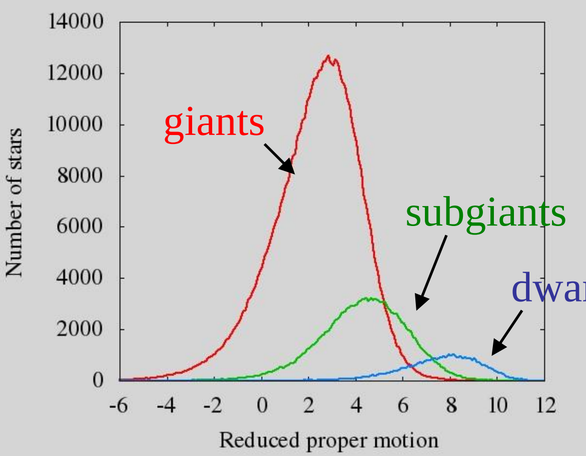
RPM

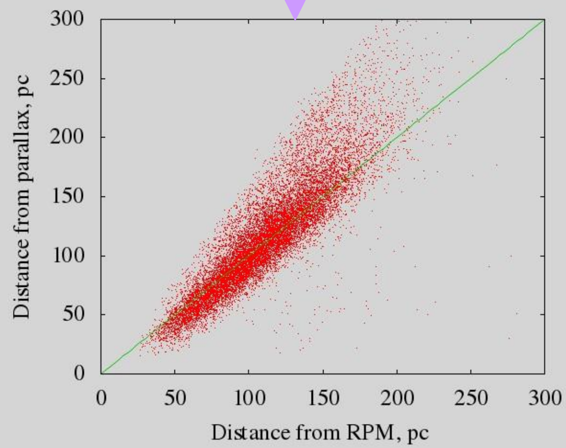
$RPM(K_S) = K_S + 5 + 5\lg(\mu),$

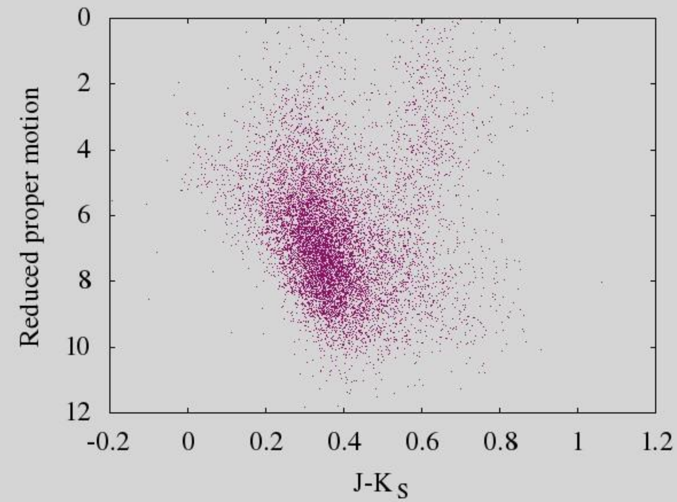
where $\mu = ((\mu_\alpha \cos_\delta)^2 + \mu_\delta)^{1/2}$

$\lg(r) = (M_{K_S} - K_S + 5)/5$









$\sigma_J , \sigma_K < 0.05$ mag

Proper motions were corrected for the differential rotation of the Galactic disk and Solar motion

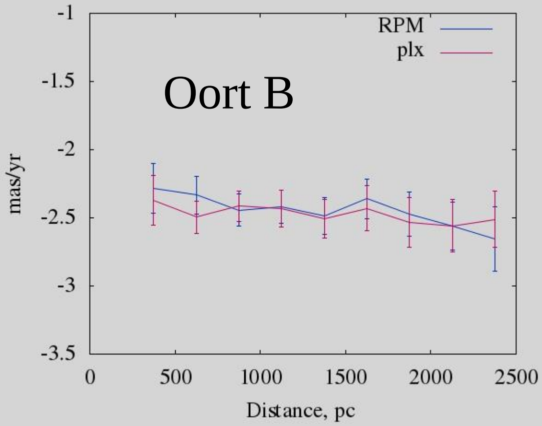
$RPM(K_S) > 8$

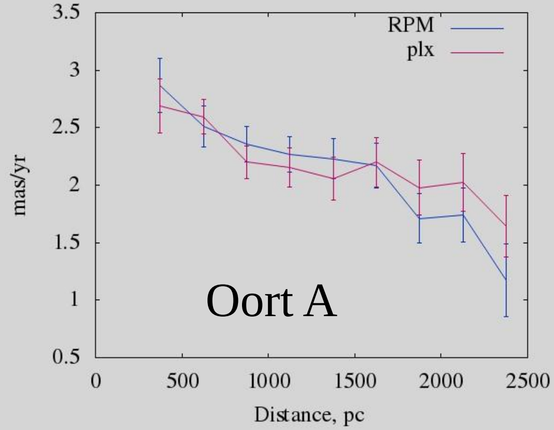
This sample of red dwarfs is not suitable for determination of kinematic parameters. Within 500 pc the **Oort B** component is **insignificant**.

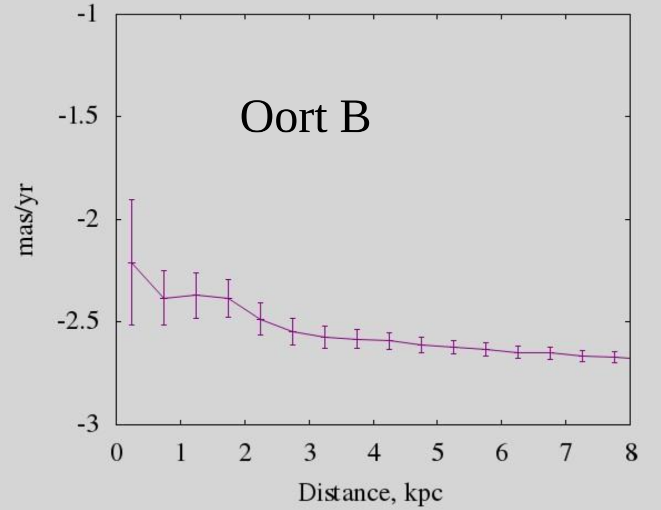
Reduced proper motions (RPM) and selection of **red giants**

Parameters form the **PMA proper motions** depending on distance derived from **parallaxes** and **RPM**

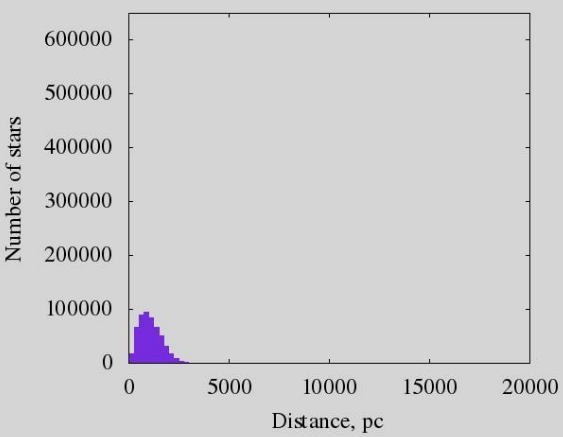
RPM(K_S) < 2

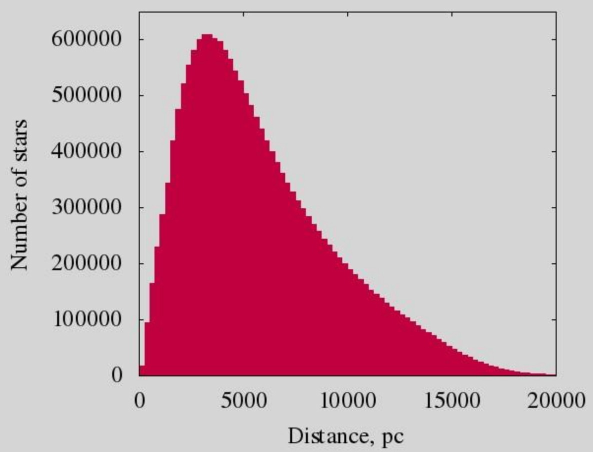


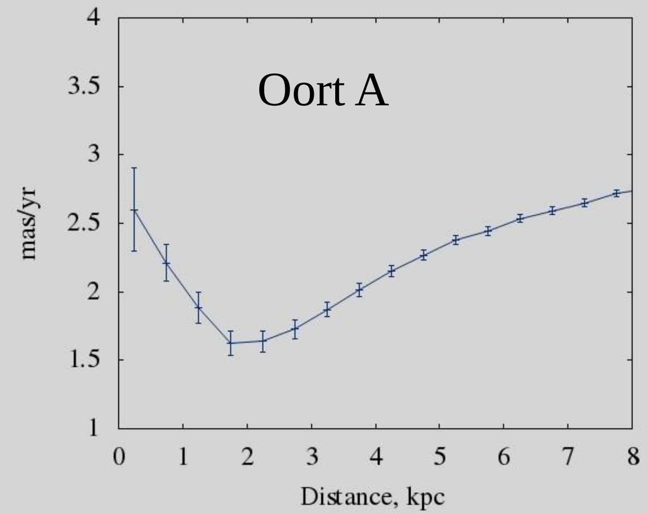




Number of red giants depending on distance for common with TGAS and full PMA samples.







Conclusions

- *There are **systematic differences** between proper motions of the PMA and TGAS catalogues.*
- *It is **hard** to establish which of these systems is inertial. We study this problem.*
- ***The PMA catalogue allows to investigate the Galactic stellar velocity field up to large distances.***
- *The kinematic parameters of the Galaxy were derived from the PMA and TGAS data.*
- *Using the decomposition into VSH the Oort B and Oort A were derived up to large distances.*
- *We confirmed conclusions derived by other authors that the O-M model is **incomplete**.*
- *There are **several extra-model** constituents in the observed stellar velocity field.*


Thank you for your attention!

Thoroidal harmonics (Mignard & Klioner, 2012)

Harm.	Mult. coef.	Components $\boldsymbol{e}_\alpha$	$\boldsymbol{e}_\delta$
$\boldsymbol{T}_{10}$	$\frac{1}{2}\sqrt{\frac{3}{2\pi}}$	$\cos\delta$	0
$\boldsymbol{T}_{11}$	$\frac{1}{4}\sqrt{\frac{3}{\pi}}$	$\sin\delta\,(\cos\alpha + \imath\sin\alpha)$	$-\sin\alpha + \imath\cos\alpha$
$\boldsymbol{T}_{20}$	$\frac{1}{4}\sqrt{\frac{15}{2\pi}}$	$\sin 2\delta$	0
$\boldsymbol{T}_{21}$	$\frac{1}{4}\sqrt{\frac{5}{\pi}}$	$-\cos 2\delta\,(\cos\alpha + \imath\sin\alpha)$	$-\sin\delta\,(\sin\alpha - \imath\cos\alpha)$
$\boldsymbol{T}_{22}$	$\frac{1}{8}\sqrt{\frac{5}{\pi}}$	$-\sin 2\delta\,(\cos 2\alpha + \imath\sin 2\alpha)$	$2\cos\delta\,(\sin 2\alpha - \imath\cos 2\alpha)$
$\boldsymbol{T}_{30}$	$\frac{1}{8}\sqrt{\frac{21}{\pi}}$	$\cos\delta\,(5\sin^2\delta - 1)$	0
$\boldsymbol{T}_{31}$	$\frac{1}{16}\sqrt{\frac{7}{\pi}}$	$\sin\delta\,(15\sin^2\delta - 11)\,(\cos\alpha + \imath\sin\alpha)$	$-(5\sin^2\delta - 1)\,(\sin\alpha - \imath\cos\alpha)$
$\boldsymbol{T}_{32}$	$\frac{1}{8}\sqrt{\frac{35}{2\pi}}$	$-\cos\delta\,(3\sin^2\delta - 1)\,(\cos 2\alpha + \imath\sin 2\alpha)$	$\sin 2\delta\,(\sin 2\alpha - \imath\cos 2\alpha)$
$\boldsymbol{T}_{33}$	$\frac{1}{16}\sqrt{\frac{105}{\pi}}$	$\cos^2\delta\sin\delta\,(\cos 3\alpha + \imath\sin 3\alpha)$	$-\cos^2\delta\,(\sin 3\alpha - \imath\cos 3\alpha)$
$\boldsymbol{T}_{40}$	$\frac{3}{16}\sqrt{\frac{5}{\pi}}$	$\sin 2\delta\,(7\sin^2\delta - 3)$	0
$\boldsymbol{T}_{41}$	$\frac{3}{16}\sqrt{\frac{1}{\pi}}$	$(28\sin^4\delta - 27\sin^2\delta + 3)\,(\cos\alpha + \imath\sin\alpha)$	$-\sin\delta\,(7\sin^2\delta - 3)\,(\sin\alpha - \imath\cos\alpha)$
$\boldsymbol{T}_{42}$	$\frac{3}{16}\sqrt{\frac{2}{\pi}}$	$-\sin 2\delta\,(7\sin^2\delta - 4)\,(\cos 2\alpha + \imath\sin 2\alpha)$	$\cos\delta\,(7\sin^2\delta - 1)\,(\sin 2\alpha - \imath\cos 2\alpha)$
$\boldsymbol{T}_{43}$	$\frac{3}{16}\sqrt{\frac{7}{\pi}}$	$\cos^2\delta\,(4\sin^2\delta - 1)\,(\cos 3\alpha + \imath\sin 3\alpha)$	$-3\cos^2\delta\sin\delta\,(\sin 3\alpha - \imath\cos 3\alpha)$
$\boldsymbol{T}_{44}$	$\frac{3}{8}\sqrt{\frac{7}{2\pi}}$	$-\cos^3\delta\sin\delta\,(\cos 4\alpha + \imath\sin 4\alpha)$	$\cos^3\delta\,(\sin 4\alpha - \imath\cos 4\alpha)$

Spheroidal harmonics (Mignard & Klioner, 2012)

Harm.	Mult. coef.	Components $\boldsymbol{e}_\alpha$	$\boldsymbol{e}_\delta$
S_{10}	$\frac{1}{2}\sqrt{\frac{3}{2\pi}}$	0	$\cos\delta$
S_{11}	$\frac{1}{4}\sqrt{\frac{3}{\pi}}$	$\sin\alpha - \imath\cos\alpha$	$\sin\delta\,(\cos\alpha + \imath\sin\alpha)$
S_{20}	$\frac{1}{4}\sqrt{\frac{15}{2\pi}}$	0	$\sin 2\delta$
S_{21}	$\frac{1}{4}\sqrt{\frac{5}{\pi}}$	$\sin\delta\,(\sin\alpha - \imath\cos\alpha)$	$-\cos 2\delta\,(\cos\alpha + \imath\sin\alpha)$
S_{22}	$\frac{1}{8}\sqrt{\frac{5}{\pi}}$	$-2\cos\delta\,(\sin 2\alpha - \imath\cos 2\alpha)$	$-\sin 2\delta\,(\cos 2\alpha + \imath\sin 2\alpha)$
S_{30}	$\frac{1}{8}\sqrt{\frac{21}{\pi}}$	0	$\cos\delta\,(5\sin^2\delta - 1)$
S_{31}	$\frac{1}{16}\sqrt{\frac{7}{\pi}}$	$(5\sin^2\delta - 1)\,(\sin\alpha - \imath\cos\alpha)$	$\sin\delta\,(15\sin^2\delta - 11)\,(\cos\alpha + \imath\sin\alpha)$
S_{32}	$\frac{1}{8}\sqrt{\frac{35}{2\pi}}$	$-\sin 2\delta\,(\sin 2\alpha - \imath\cos 2\alpha)$	$-\cos\delta\,(3\sin^2\delta - 1)\,(\cos 2\alpha + \imath\sin 2\alpha)$
S_{33}	$\frac{1}{16}\sqrt{\frac{105}{\pi}}$	$\cos^2\delta\,(\sin 3\alpha - \imath\cos 3\alpha)$	$\cos^2\delta\sin\delta\,(\cos 3\alpha + \imath\sin 3\alpha)$
S_{40}	$\frac{3}{16}\sqrt{\frac{5}{\pi}}$	0	$\sin 2\delta\,(7\sin^2\delta - 3)$
S_{41}	$\frac{3}{16}\sqrt{\frac{1}{\pi}}$	$\sin\delta\,(7\sin^2\delta - 3)\,(\sin\alpha - \imath\cos\alpha)$	$(28\sin^4\delta - 27\sin^2\delta + 3)\,(\cos\alpha + \imath\sin\alpha)$
S_{42}	$\frac{3}{16}\sqrt{\frac{2}{\pi}}$	$-\cos\delta\,(7\sin^2\delta - 1)\,(\sin 2\alpha - \imath\cos 2\alpha)$	$-\sin 2\delta\,(7\sin^2\delta - 4)\,(\cos 2\alpha + \imath\sin 2\alpha)$
S_{43}	$\frac{3}{16}\sqrt{\frac{7}{\pi}}$	$3\cos^2\delta\,\sin\delta\,(\sin 3\alpha - \imath\cos 3\alpha)$	$\cos^2\delta\,(4\sin^2\delta - 1)\,(\cos 3\alpha + \imath\sin 3\alpha)$
S_{44}	$\frac{3}{8}\sqrt{\frac{7}{2\pi}}$	$-\cos^3\delta\,(\sin 4\alpha - \imath\cos 4\alpha)$	$-\cos^3\delta\,\sin\delta\,(\cos 4\alpha + \imath\sin 4\alpha)$