

# The Tycho-Gaia Astrometric Solution (TGAS)

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# Outline of talk

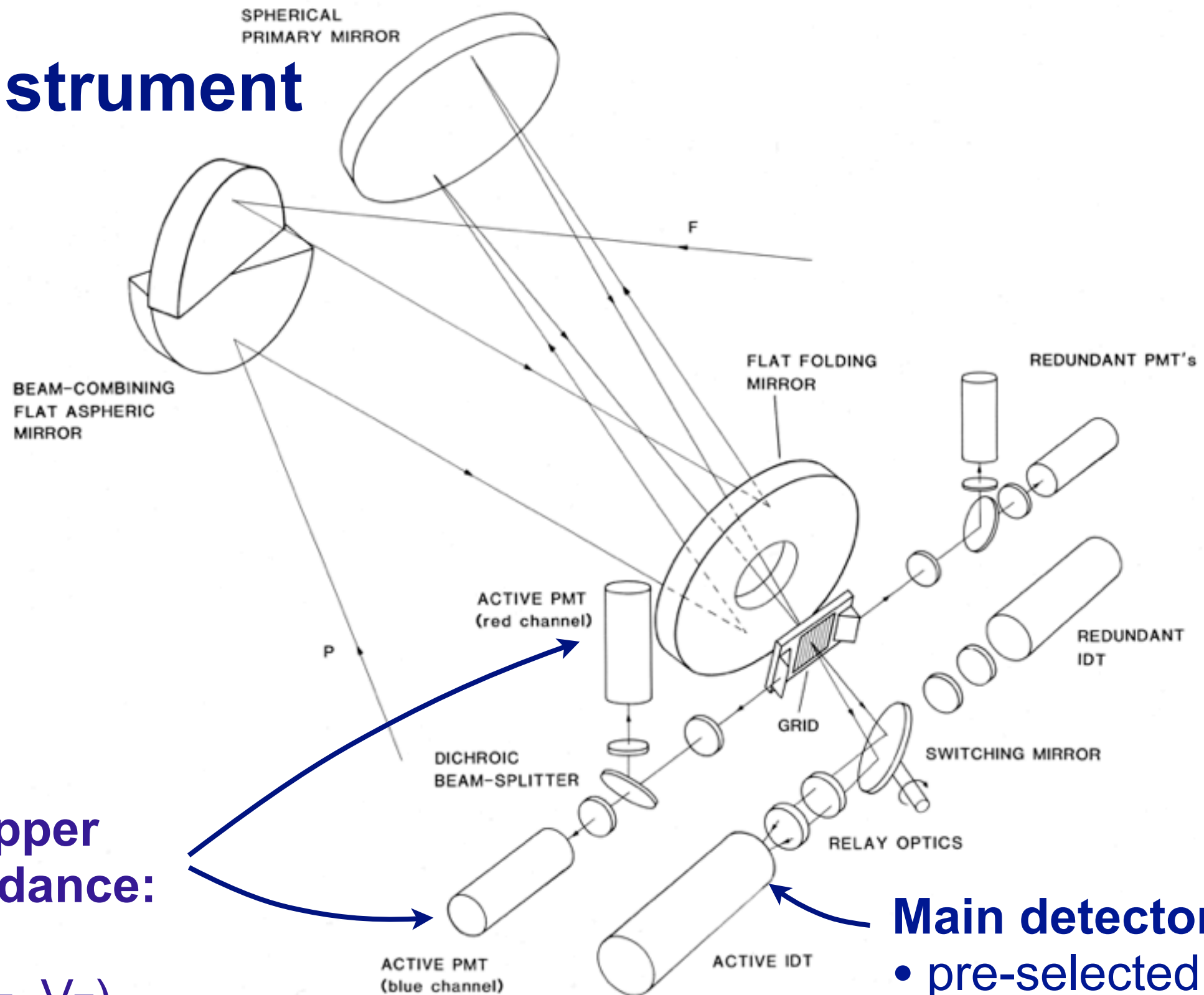
- Historical background and concept
- Reference frame of TGAS
- Uncertainties
- Correlations
- Systematics
- Conclusions

# The Hipparcos instrument



## Auxiliary starmapper detectors for guidance:

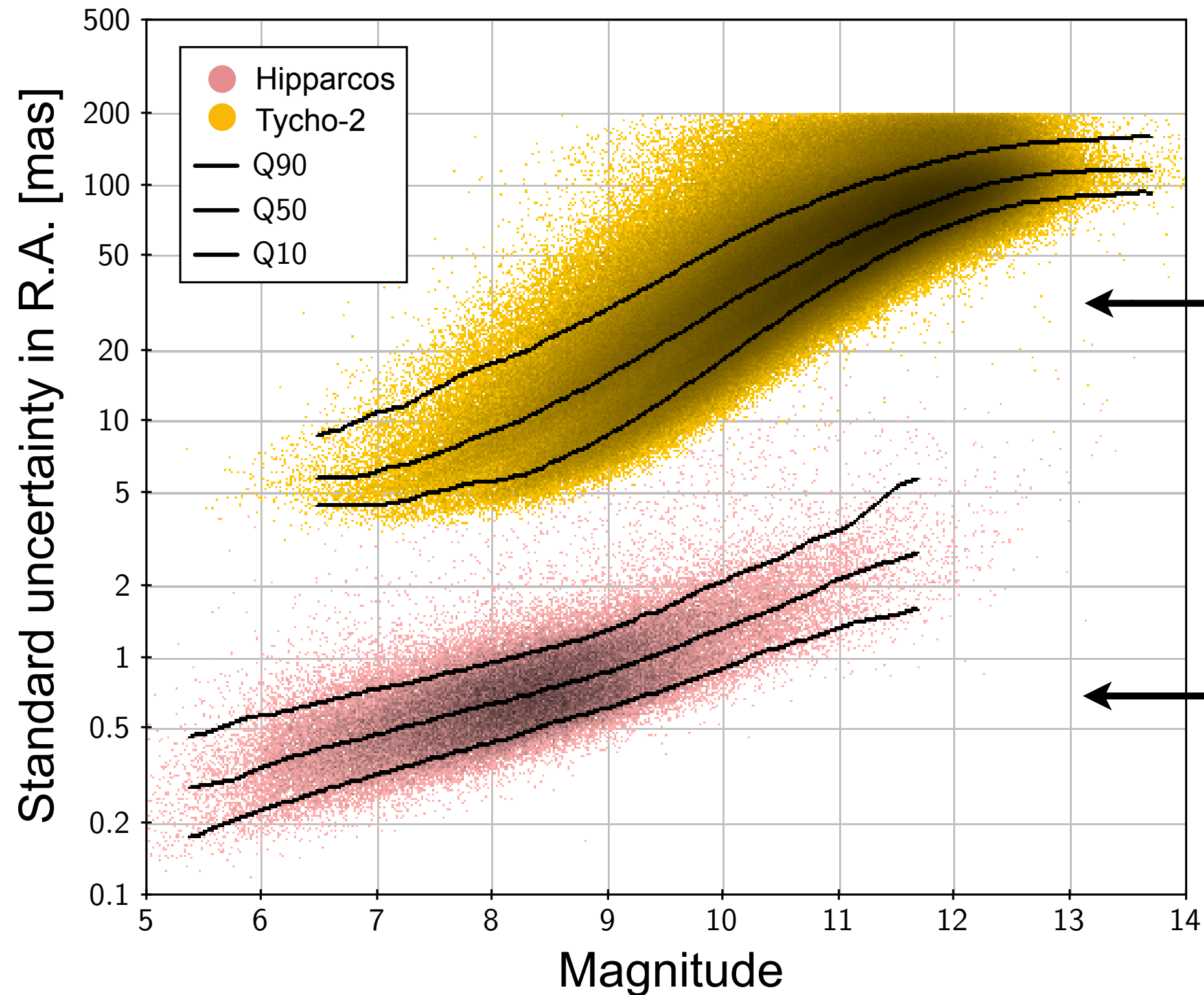
- no pre-selection
- two channels ( $B_T$ ,  $V_T$ )
- Tycho/Tycho-2 Catalogue



## Main detector:

- pre-selected stars
- one channel ( $H_p$ )
- Hipparcos Catalogue

# Hipparcos and Tycho-2: Positional errors at 1991.25



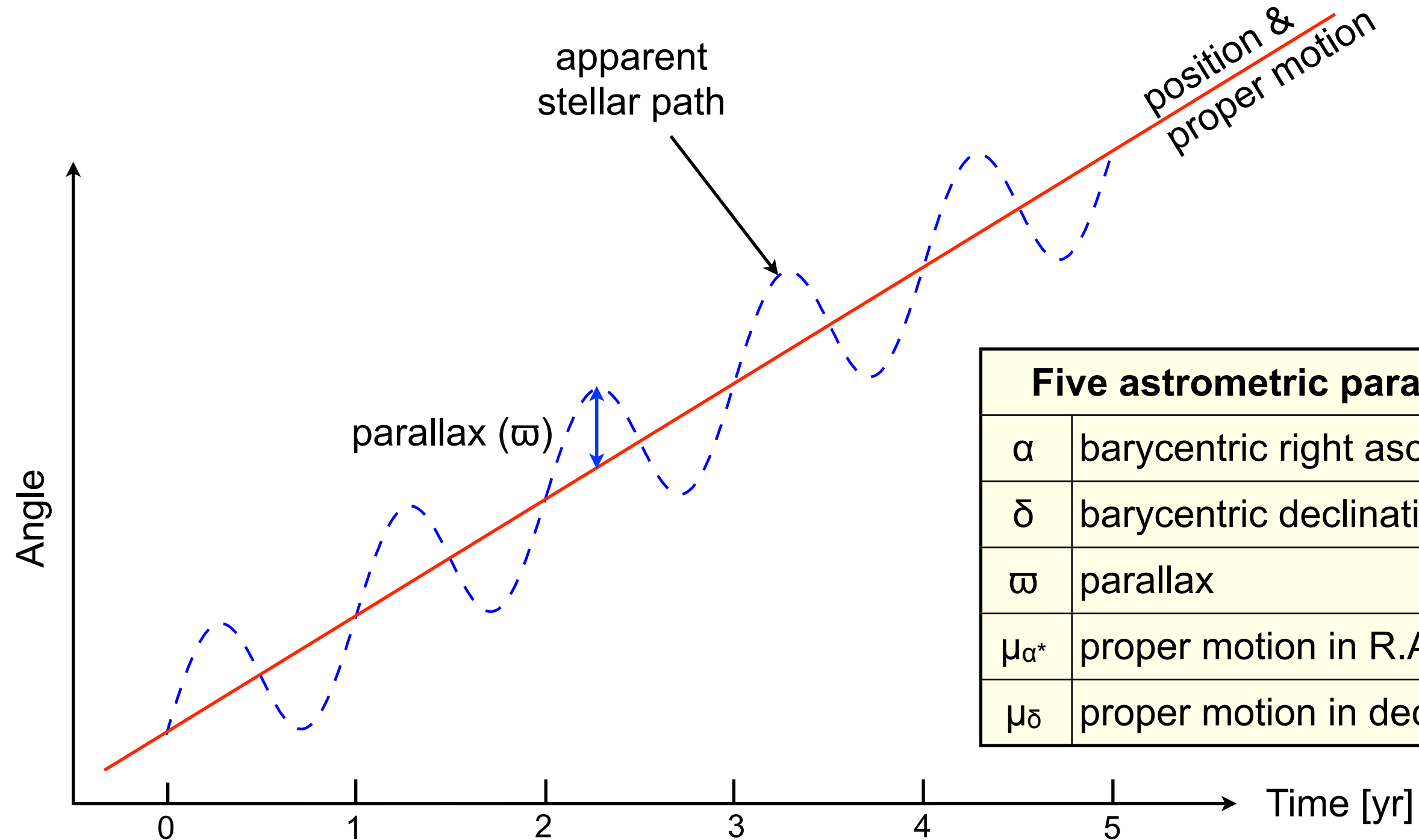
Tycho-2 Catalogue  
2.5 million stars  
(Høg et al. 2000)

Hipparcos Catalogue  
118,000 stars  
(van Leeuwen 2007)

# From HTPM to TGAS

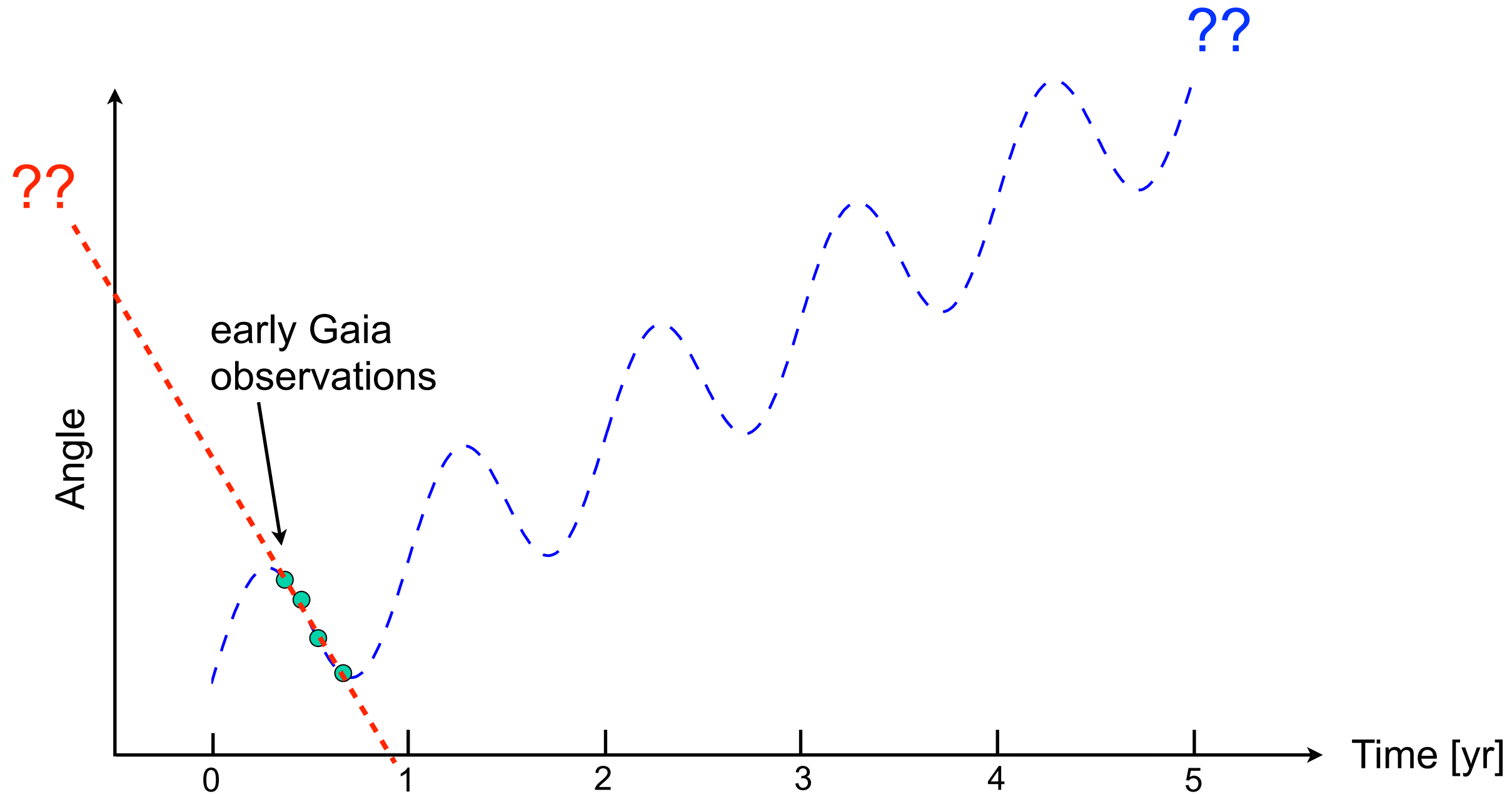
- In 2009 François Mignard proposed the *Hundred Thousand Proper Motions (HTPM)* project as an early “appetizer” for things to come from the Gaia mission
- Basic idea: to get a 10-fold improvement of the Hipparcos proper motions from early Gaia observations of Hipparcos stars, using the >20 yr epoch difference
- HTPM was adopted as part of Gaia DR1 in the official data release scenario
- A natural extension of HTPM is to use Tycho-2 positions as well  
→ *Tycho-Gaia Astrometric Solution (TGAS)* (Michalik et al., A&A 574, A115, 2015)
- TGAS allowed us to *verify Gaia’s astrometric performance* and *improve the calibration models* about 1 year earlier than otherwise possible

# Astrometric model for a single star



Five astrometric parameters per star	
$\alpha$	barycentric right ascension at 2015.0
$\delta$	barycentric declination at 2015.0
$\varpi$	parallax
$\mu_{\alpha^*}$	proper motion in R.A. ( $\times \cos \delta$ )
$\mu_{\delta}$	proper motion in declination

# Degeneracy for <1 yr of observations





# TGAS: Using the old position as prior

Only *positions* were taken from Hipparcos and Tycho-2, no parallaxes or proper motions!

early Gaia observations

Starting with Gaia DR2, no prior information from Hipparcos and Tycho-2 will be used

Hipparcos  
or Tycho-2  
position  
(1991.25)

- Two distinct subsets in TGAS:
- Hip subset (~90,000 stars) using Hipparcos positions
  - Tyc subset (~2 million stars) using Tycho-2 positions



# The reference frame of Gaia DR1

The ICRS = *International Celestial Reference System* (IAU 1997) is the basis for all modern astrometric catalogues (replacing FK4, FK5, equinox B1950, J2000, etc)

ICRS is *non-rotating* with respect to distant quasars – orientation defined by radio (VLBI) observations of ~300 quasars

Before Gaia, the *Hipparcos Catalogue* was the best *optical* realisation of the ICRS

Both Hipparcos and TGAS are nominally on ICRS

But we find that the *Hipparcos Reference Frame rotates wrt ICRS by 0.24 mas/yr* :

$$\mu_{\alpha^*,\text{ICRS}} = \mu_{\alpha^*,\text{HIP}} - 0.126 \sin \delta \cos \alpha + 0.185 \sin \delta \sin \alpha - 0.076 \cos \delta$$

$$\mu_{\delta,\text{ICRS}} = \mu_{\delta,\text{HIP}} + 0.126 \sin \alpha + 0.185 \cos \alpha$$

# Uncertainties in TGAS

Five astrometric parameters per star:

$$\alpha, \delta, \varpi, \mu_{\alpha^*}, \mu_{\delta}$$

Five standard uncertainties:

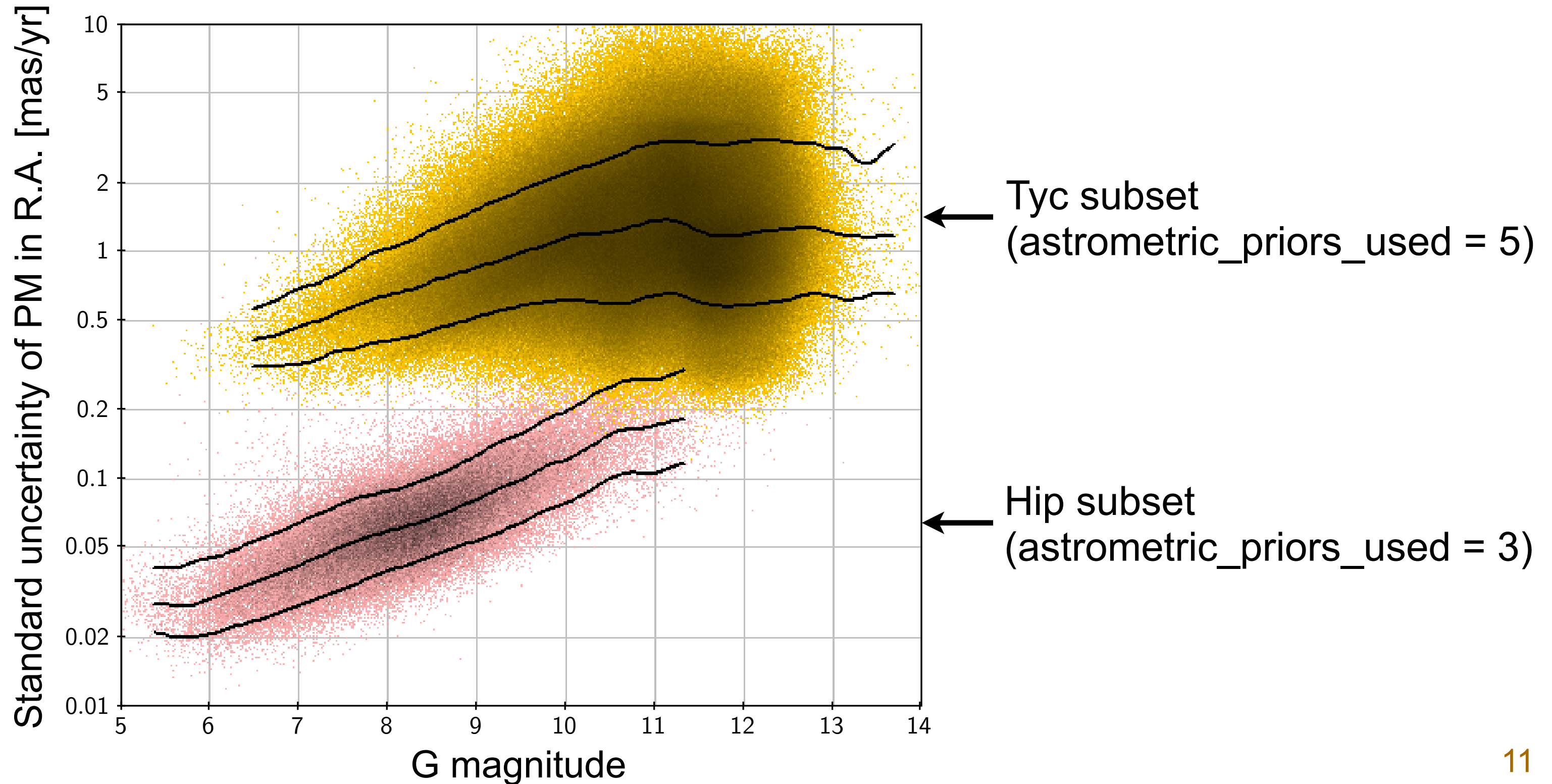
$$\sigma_{\alpha^*}, \sigma_{\delta}, \sigma_{\varpi}, \sigma_{\mu_{\alpha^*}}, \sigma_{\mu_{\delta}}$$

**Gaia Archive:**

**ra, dec, parallax, pmra, pmdec**

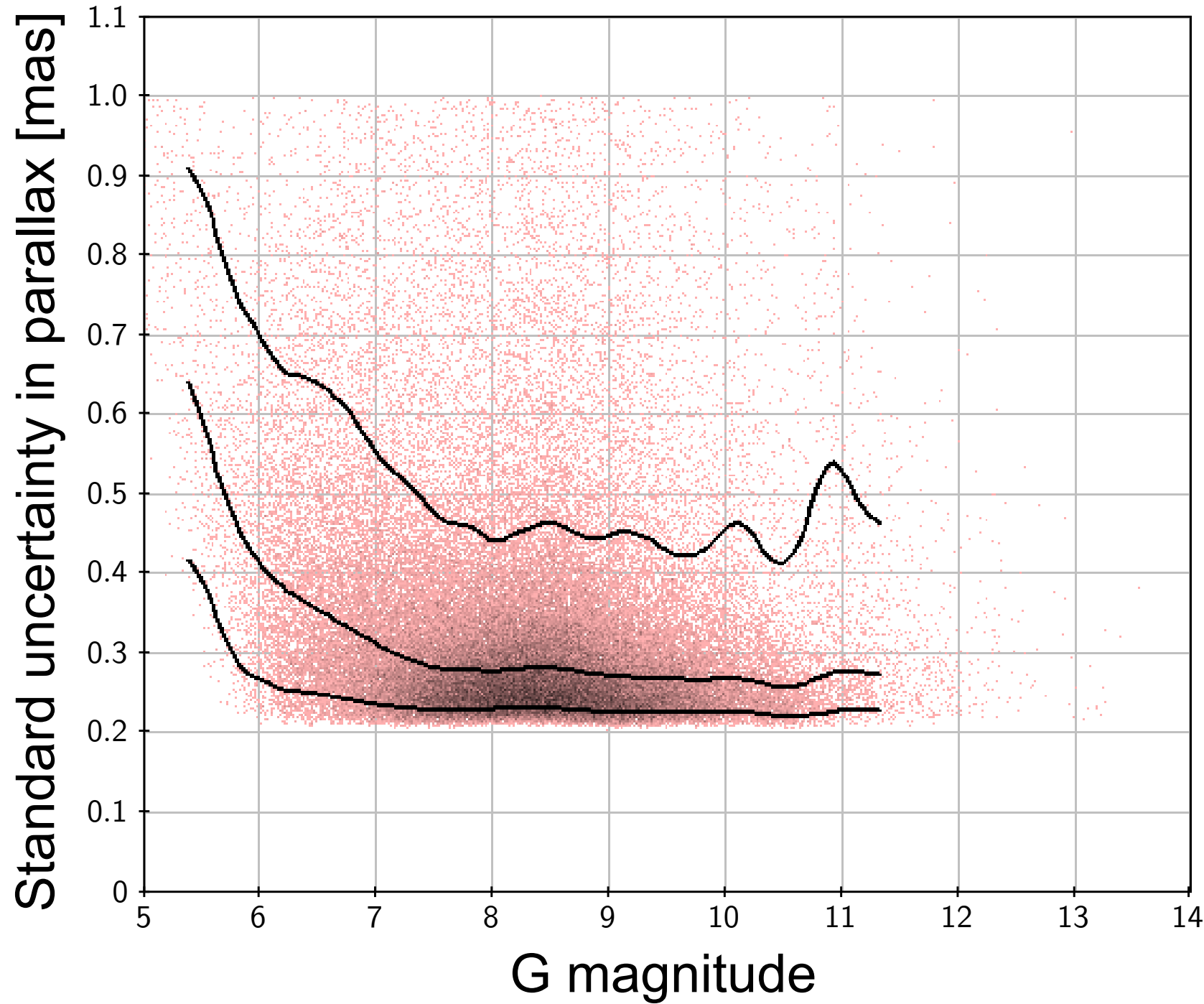
**ra\_error, dec\_error, ...**

# Standard uncertainty in proper motion versus magnitude

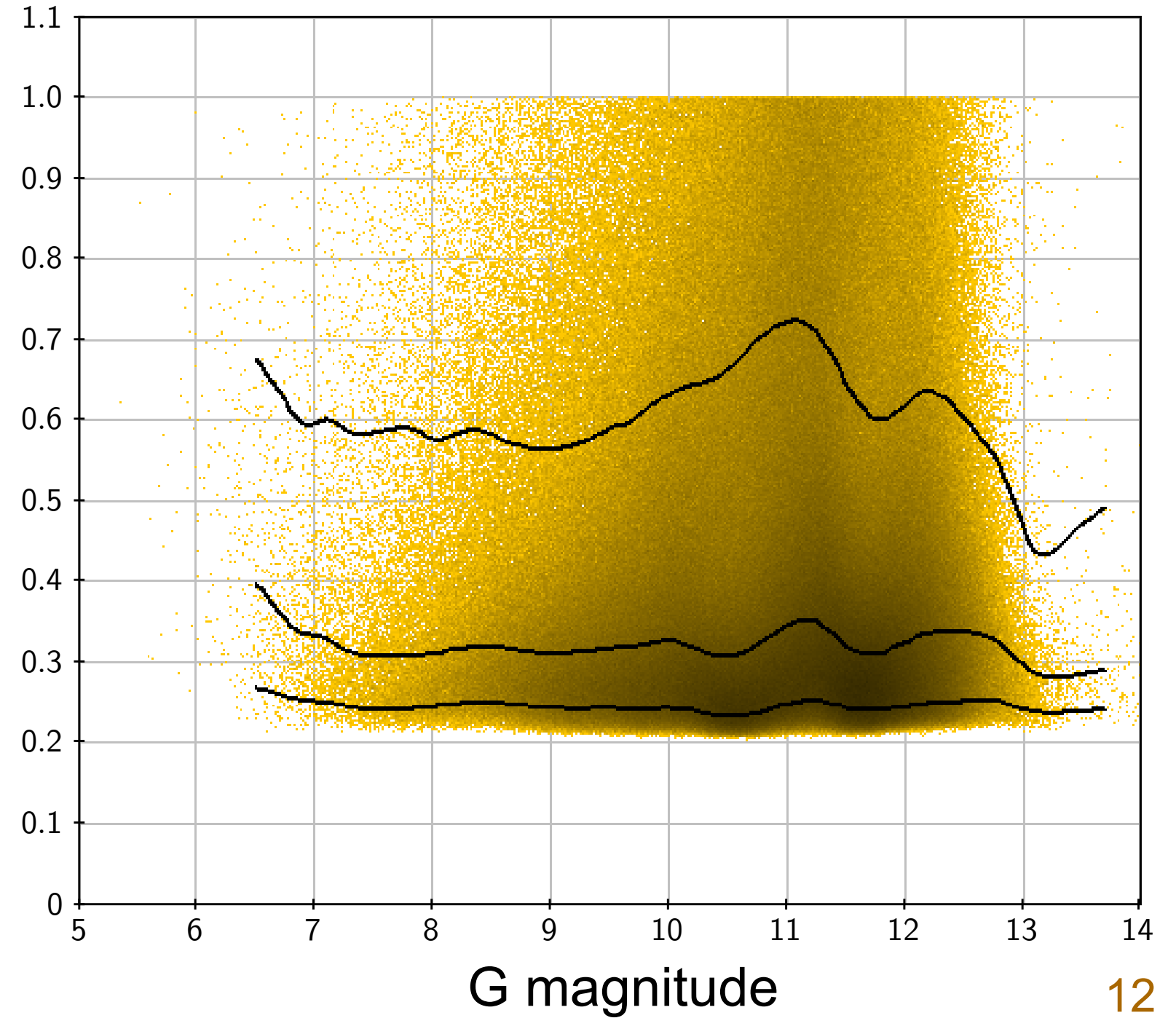


# Standard uncertainty in parallax versus magnitude

Hip subset



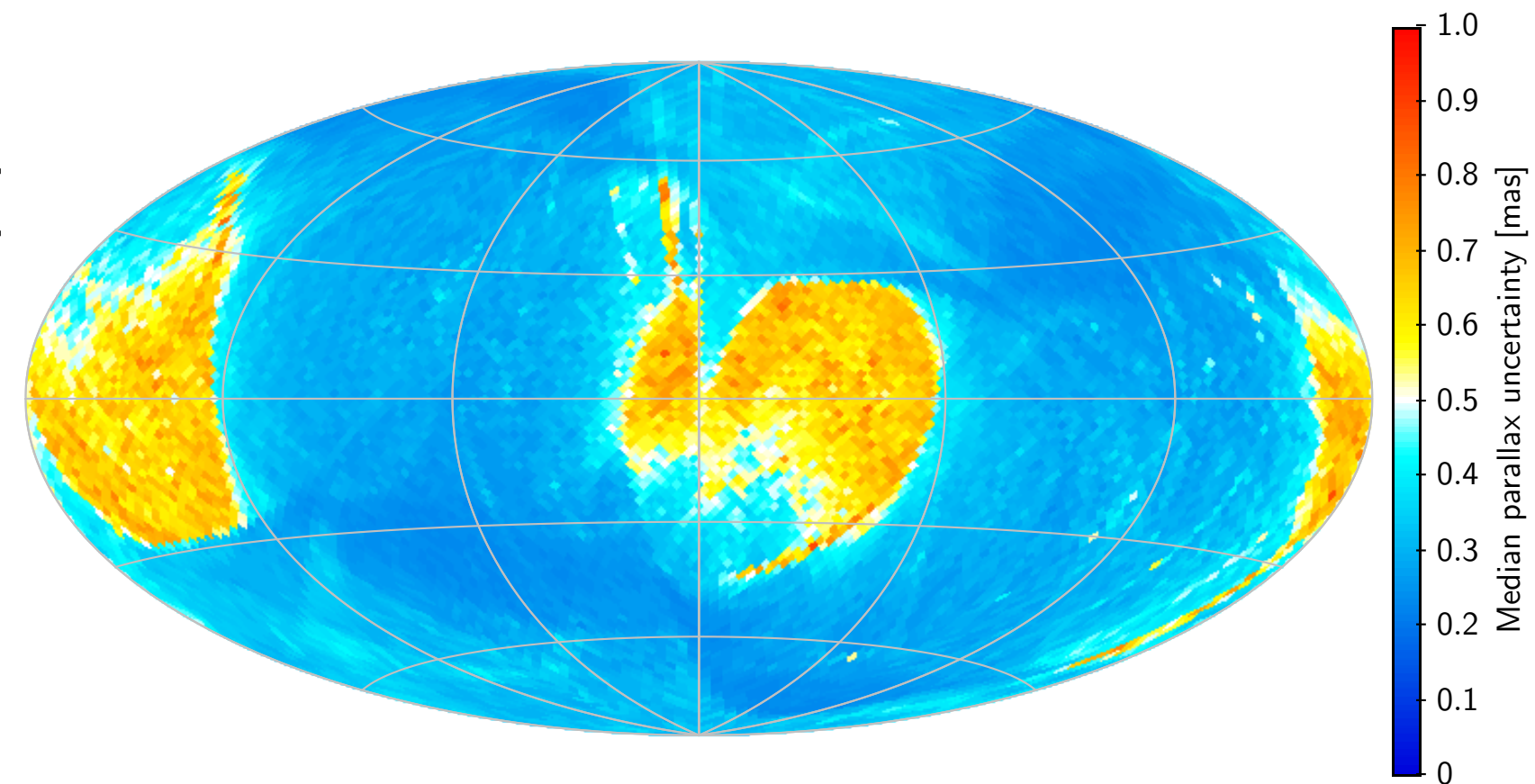
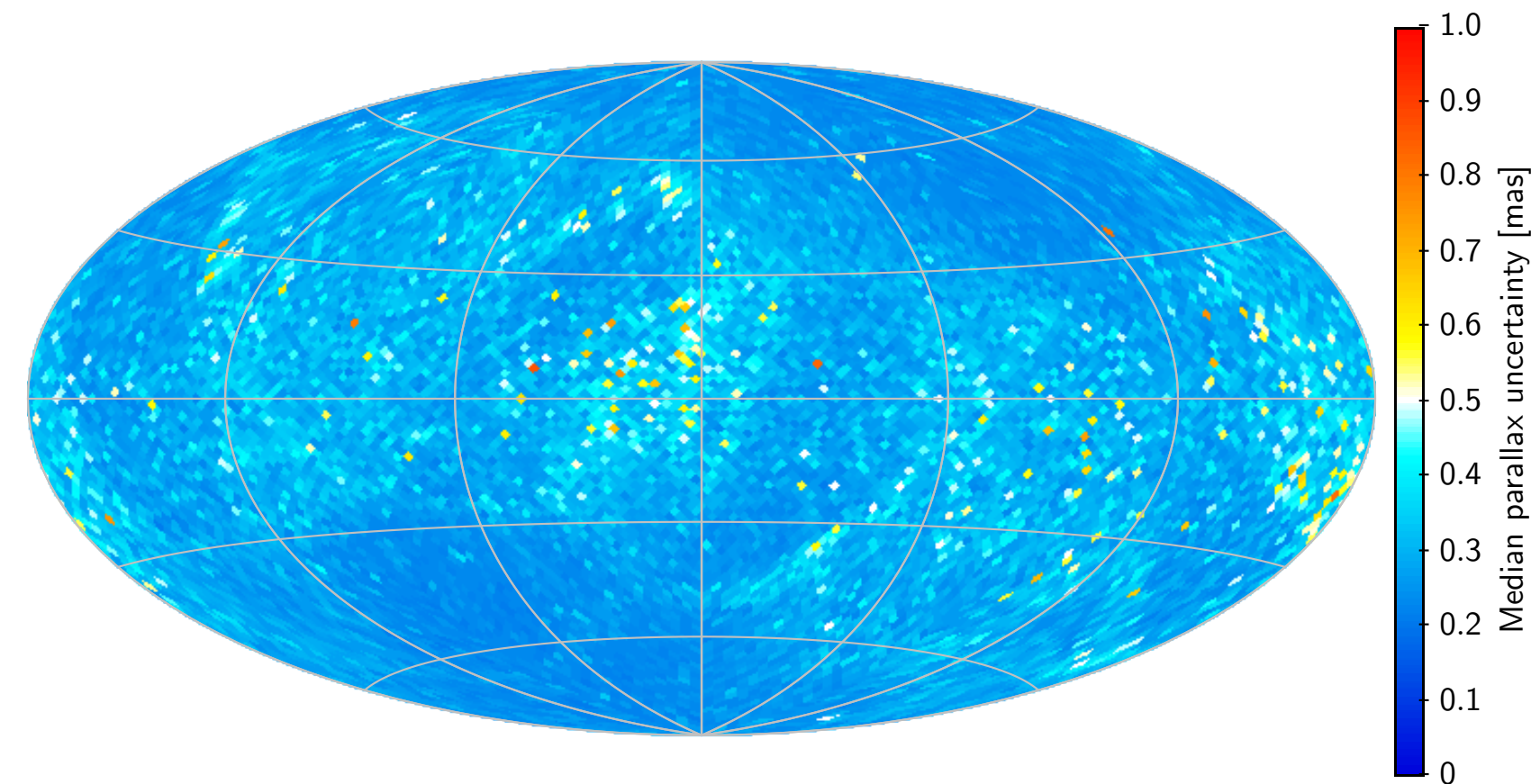
Tyc subset



# Median standard uncertainty in parallax versus position

Hip subset

Tyc subset



# Correlations in TGAS

Five astrometric parameters per star:

$$\alpha, \delta, \varpi, \mu_{\alpha^*}, \mu_{\delta}$$

Five standard uncertainties:

$$\sigma_{\alpha^*}, \sigma_{\delta}, \sigma_{\varpi}, \sigma_{\mu_{\alpha^*}}, \sigma_{\mu_{\delta}}$$

Ten correlation coefficients ( $-1 < \rho < 1$ ):

$$\begin{array}{cccc} \rho(\alpha, \delta), & \rho(\alpha, \varpi), & \rho(\alpha, \mu_{\alpha^*}), & \rho(\alpha, \mu_{\delta}), \\ & \rho(\delta, \varpi), & \rho(\delta, \mu_{\alpha^*}), & \rho(\delta, \mu_{\delta}), \\ & & \rho(\varpi, \mu_{\alpha^*}), & \rho(\varpi, \mu_{\delta}), \\ & & & \rho(\mu_{\alpha^*}, \mu_{\delta}) \end{array}$$

Gaia Archive:

ra, dec, parallax, pmra, pmdec

ra\_error, dec\_error, ...

ra\_dec\_corr, ...

# Correlations matter!

- Correlation coefficients ( $\rho$ ) are needed for error propagation involving more than one astrometric parameter, e.g.:

$$v_T = 4.7405 \mu / \varpi \quad \Rightarrow \quad \left( \frac{\sigma_{v_T}}{v_T} \right)^2 = \left( \frac{\sigma_\mu}{\mu} \right)^2 + \left( \frac{\sigma_\varpi}{\varpi} \right)^2 - \underbrace{2\rho(\varpi, \mu) \left( \frac{\sigma_\mu}{\mu} \right) \left( \frac{\sigma_\varpi}{\varpi} \right)}_{\text{neglecting this can give very wrong } \sigma_{v_T}}$$

- Also when estimating model parameters (chi-square fitting, MLE, Bayesian analysis, ...):

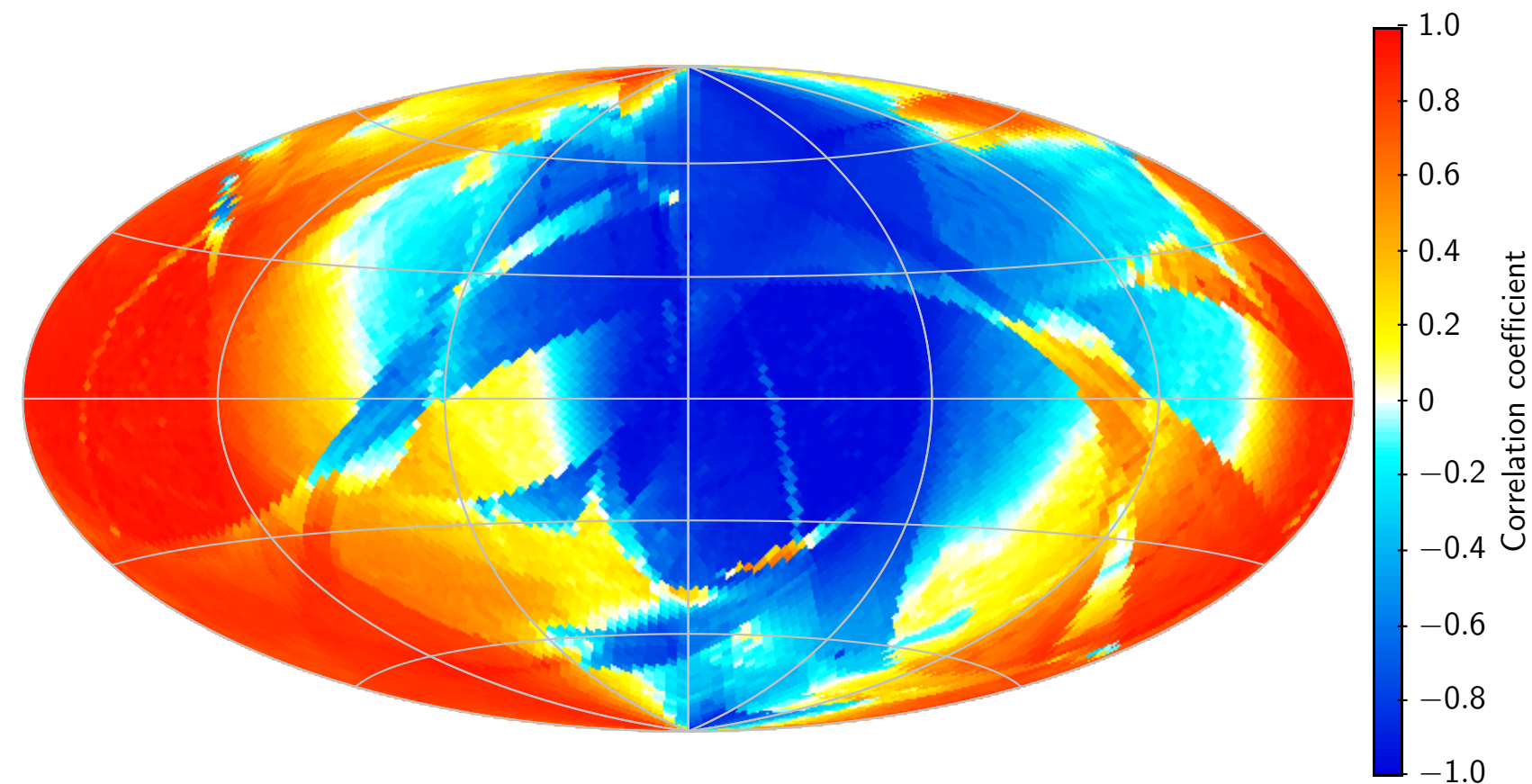
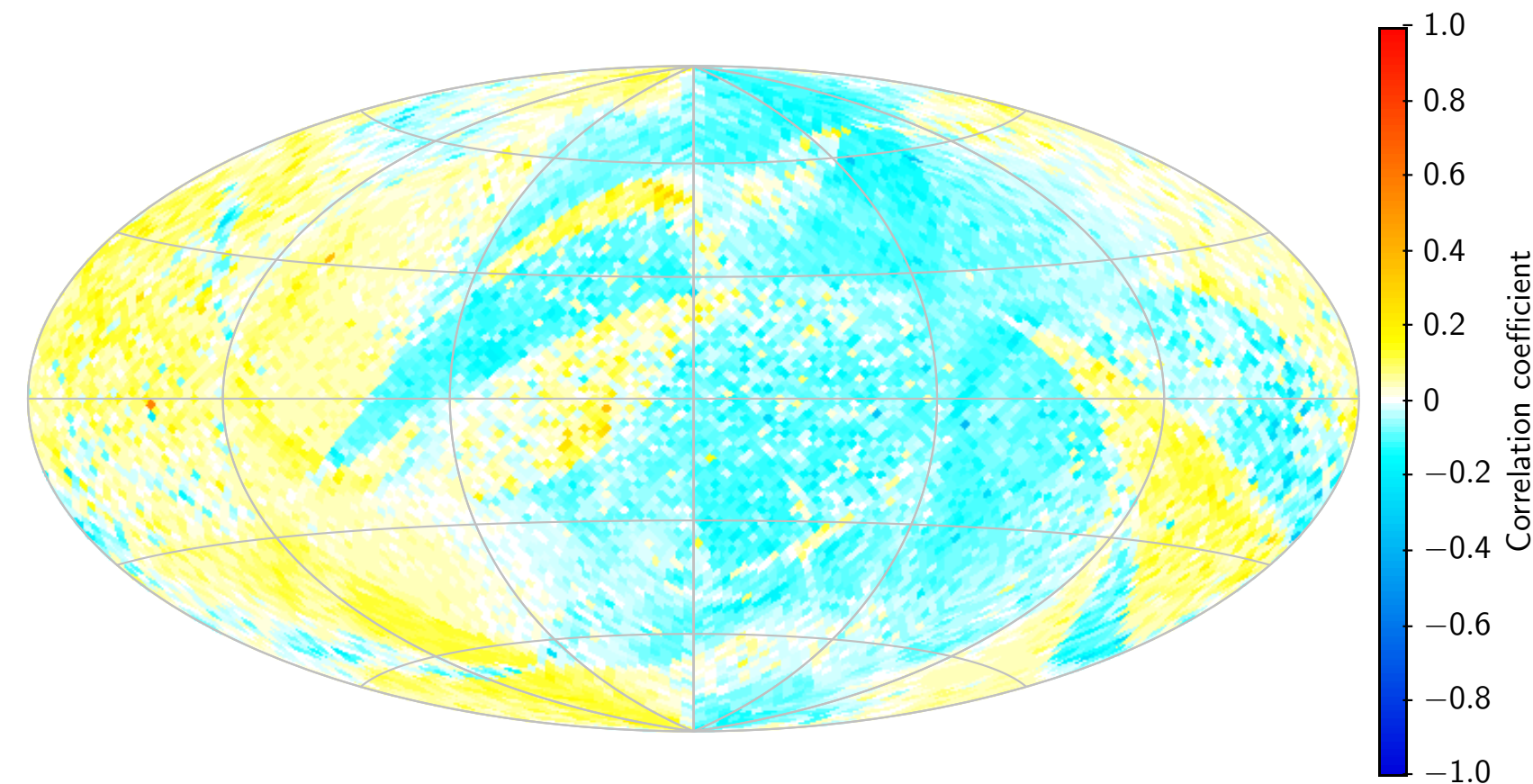
$$\chi^2 = \begin{bmatrix} \Delta\mu_{\alpha^*} & \Delta\mu_\delta & \Delta\varpi \end{bmatrix} \mathbf{C}^{-1} \begin{bmatrix} \Delta\mu_{\alpha^*} \\ \Delta\mu_\delta \\ \Delta\varpi \end{bmatrix} \quad (\mathbf{C} = \text{covariance matrix})$$



# Median correlation coefficient ( $\varpi$ , $\mu_{\alpha^*}$ ) versus position

Hip subset

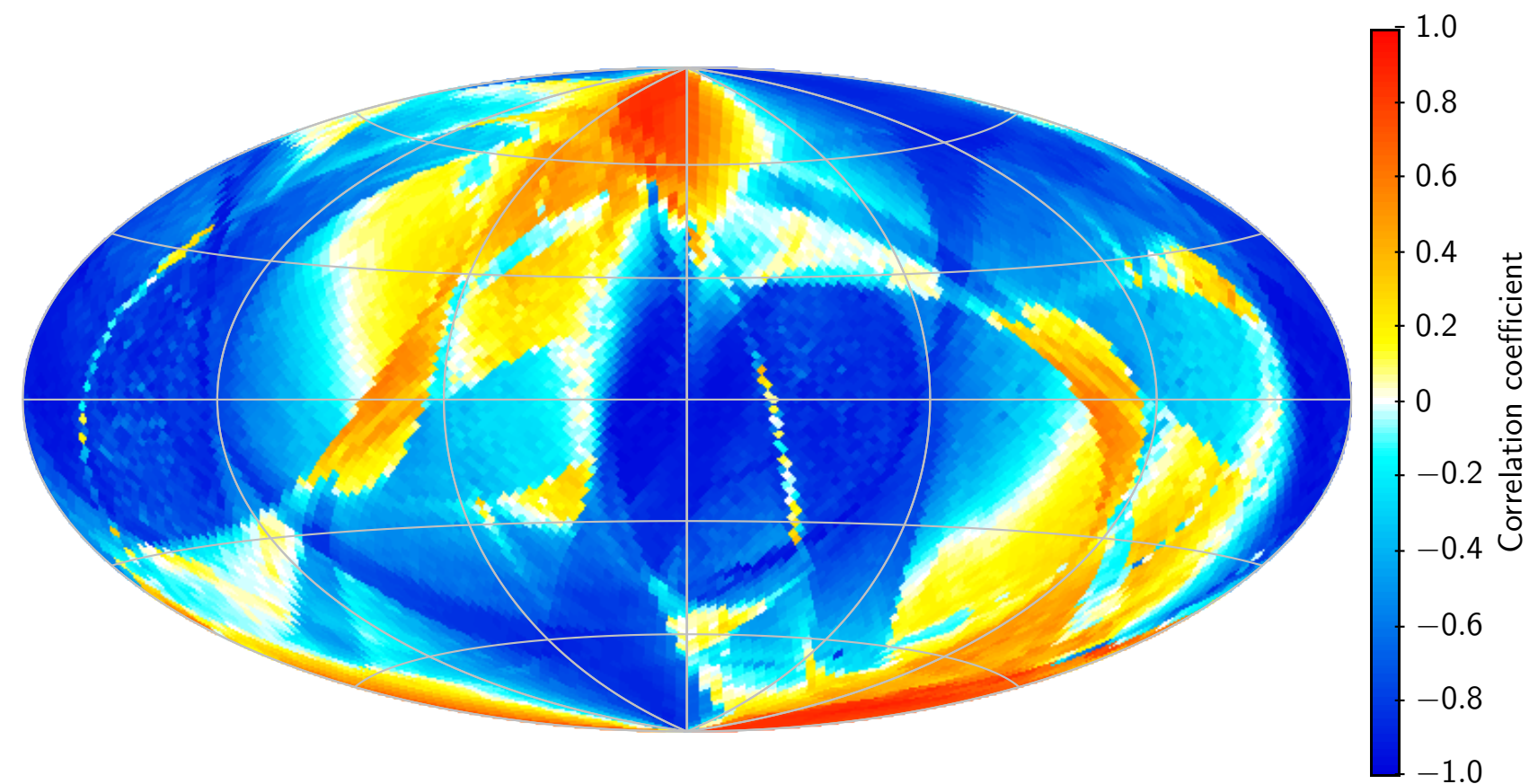
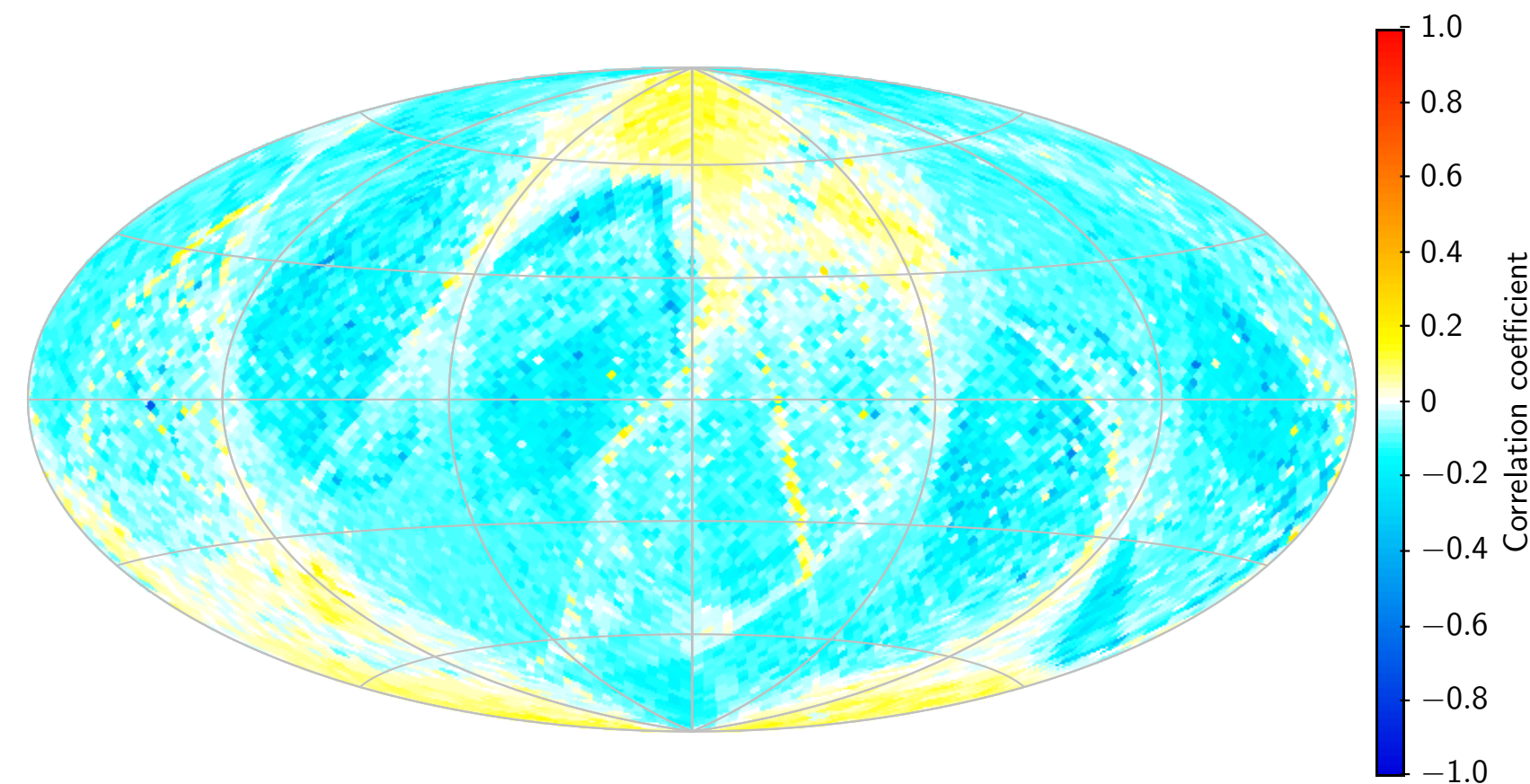
Tyc subset



# Median correlation coefficient ( $\varpi$ , $\mu_\delta$ ) versus position

Hip subset

Tyc subset

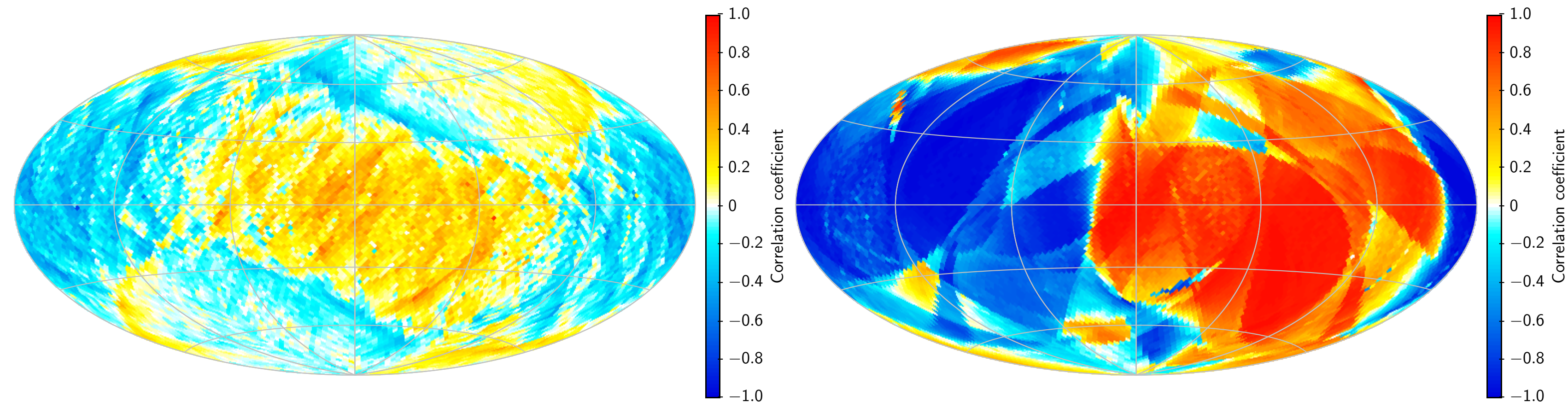




# Median correlation coefficient ( $\mu_{\alpha^*}$ , $\mu_{\delta}$ ) versus position

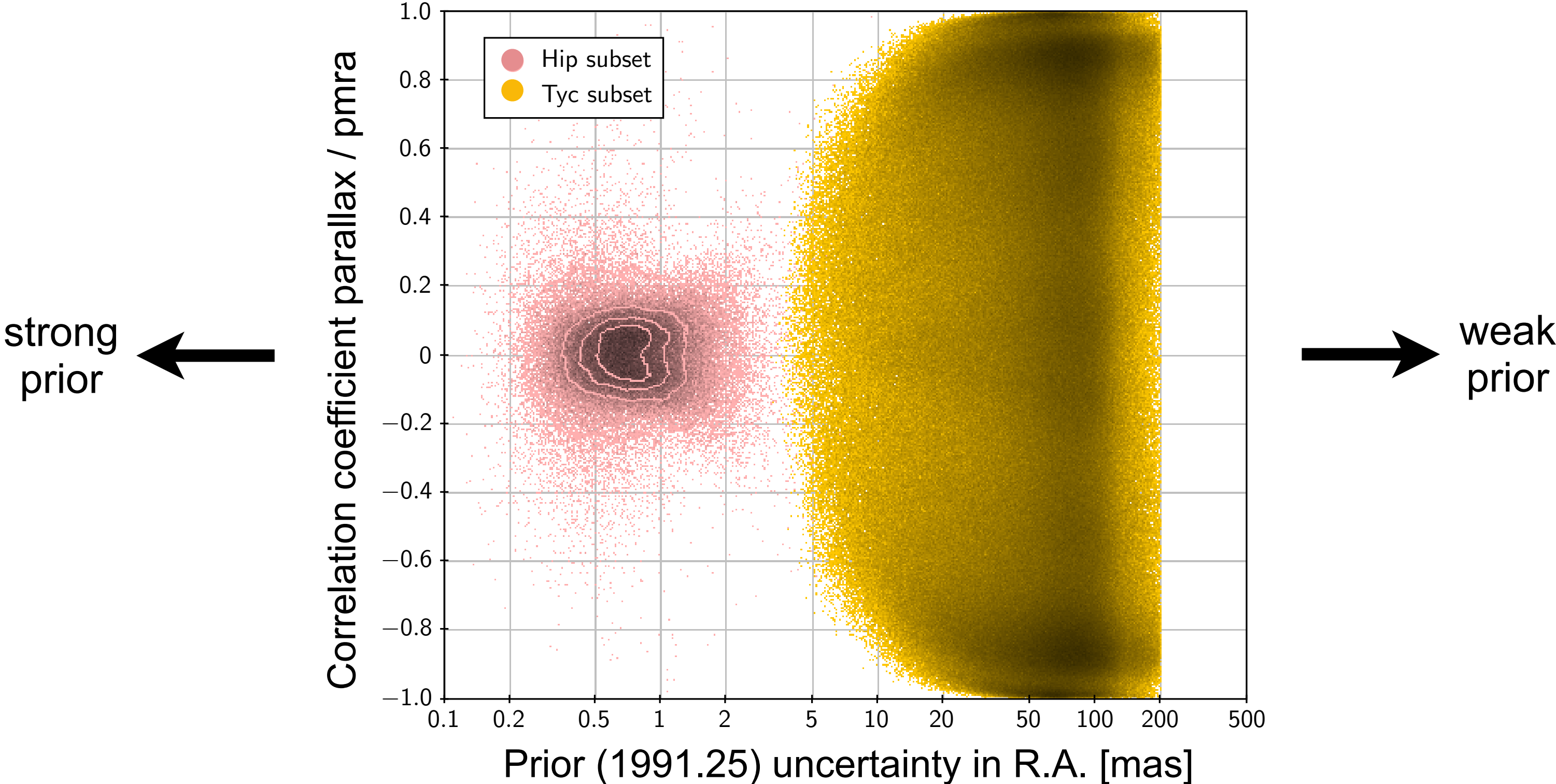
Hip subset

Tyc subset



**Correlations often approach  $\pm 1$  in the Tycho subset!**

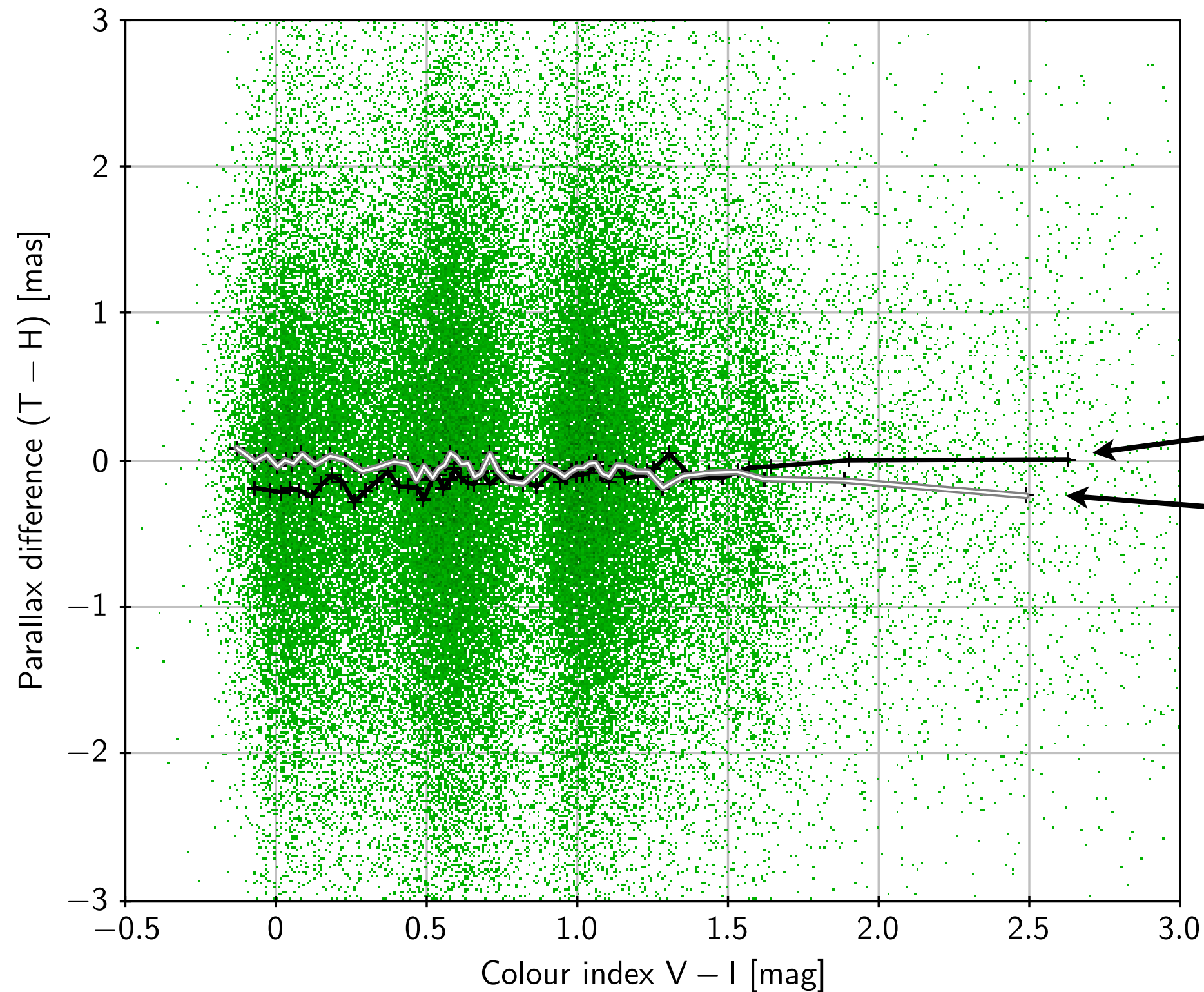
# Correlations depend on the strength of the prior



# Systematic errors in TGAS

- Systematic errors exist, and are complicated (largely unknown) functions of position, magnitude, colour, prior used, ...
- Systematics have been investigated through
  - comparison with external data (e.g. Hipparcos)

# Example: $\varpi_{\text{TGAS}} - \varpi_{\text{HIP}}$ versus colour index



Median difference for:

$\beta > 0$  (ecliptic north)

$\beta < 0$  (ecliptic south)

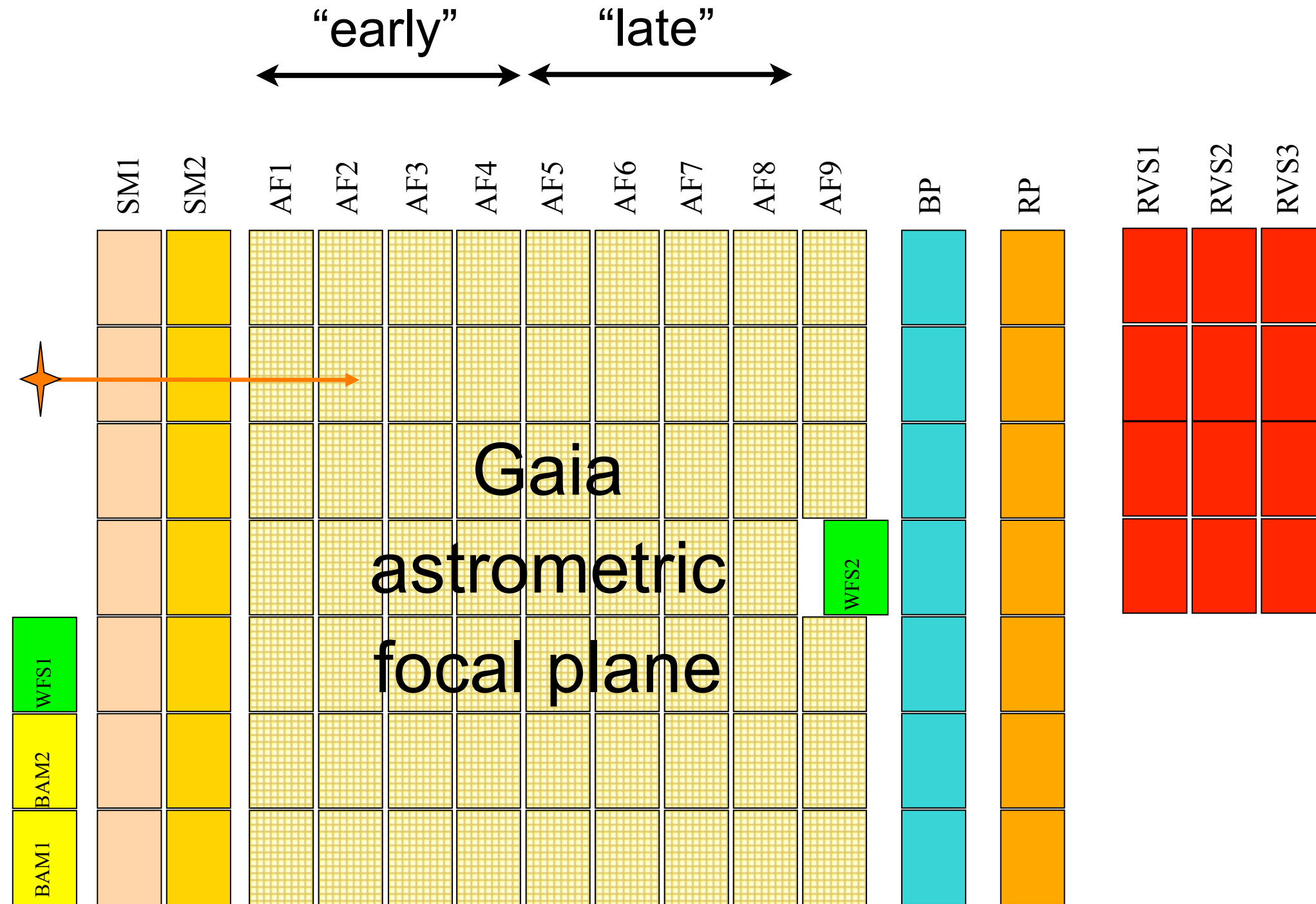
→ Colour & position dependent systematics on the level  $\pm 0.1$  mas

# Systematic errors in TGAS

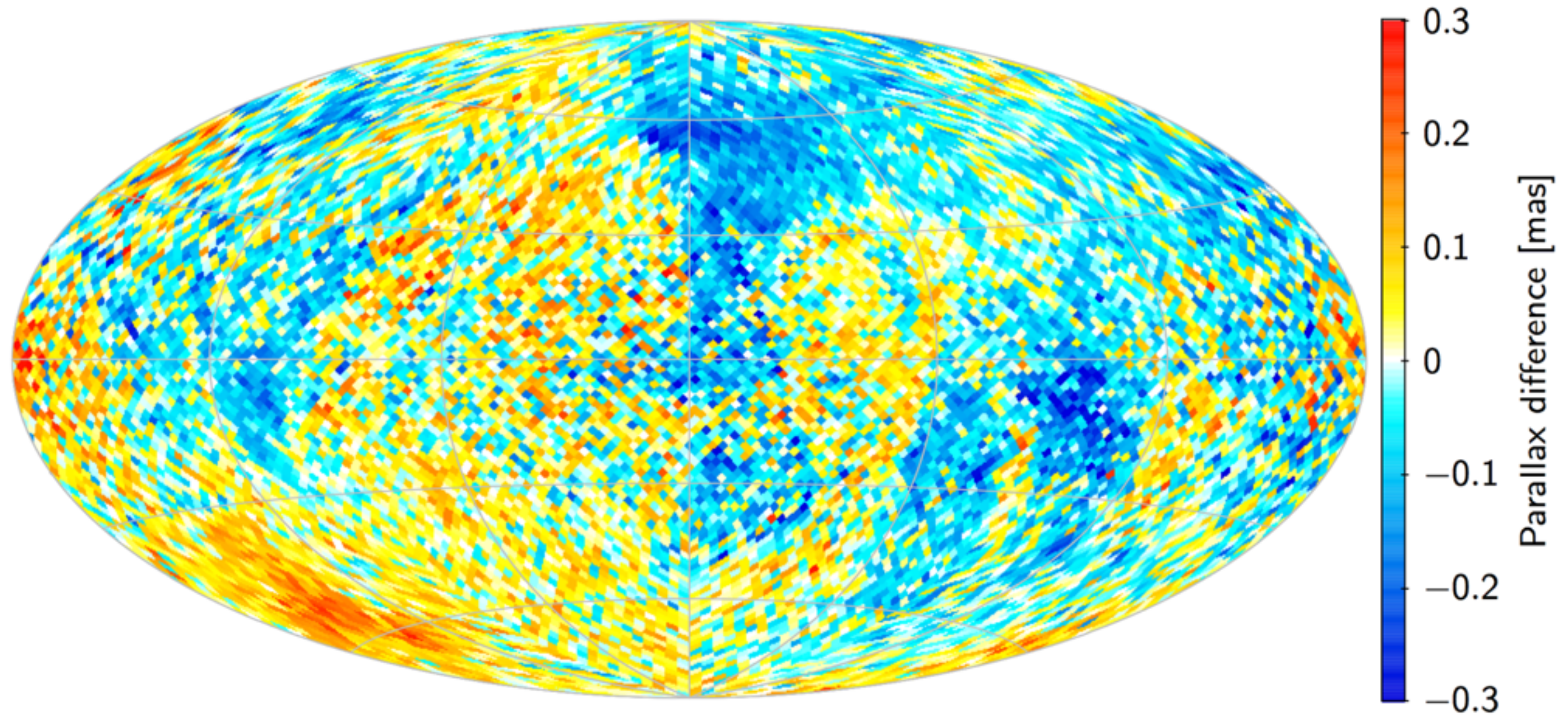
- Systematic errors exist, and are complicated (largely unknown) functions of the position, magnitude, colour, prior used, ...
- Systematics have been investigated through
  - comparison with external data (e.g. Hipparcos)
  - special validation solutions (e.g. splitting the data and making separate solutions)



# Example: Splitting the observations in two datasets



# Parallax difference between two validation solutions for “early” and “late” data





# Systematic errors in TGAS

- Systematic errors exist, and are complicated (largely unknown) functions of the position, magnitude, colour, prior used, ...
- Systematics have been investigated through
  - comparison with external data (e.g. Hipparcos)
  - special validation solutions (e.g. splitting the data and making separate solutions)
- For the parallaxes
  - a global offset of  $\pm 0.1$  mas may be present
  - there are colour dependent, spatially correlated errors of  $\pm 0.3$  mas
  - in small areas even reaching  $\pm 1$  mas
- These systematics are included in the standard uncertainties
  - see Gaia DR1 papers (A&A 595, 2016) for details

# Conclusions

- The reference frame of Gaia DR1 is not the same as the Hipparcos Reference Frame (although both are nominally on ICRS)
- TGAS consists of two subsets: `astrometric_priors_used = 3` (Hip) or `5` (Tyc), with very different properties – decisive factor is the strength of the prior
- Correlations in TGAS are very important, especially for weak priors (faint Tyc subset)
- Systematics are at the few 0.1 mas or mas/yr level, probably higher for weak priors
- Future Gaia data releases will tell us how good or bad TGAS really is!

