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# Self-consistent modelling of the Galaxy with Gaia data

# Outline

- The Standard Galaxy
- Equilibrium models, Jeans' theorem & AA variables
- Achieving self-consistency
- Recent work on equilibrium models, self-consistent & otherwise
- Flies in the ointment: non-axisymmetry and non-stationarity
- Orbit models: Schwarzschild, M2M and Torus Mapping
- Selection functions: the need for a model history

# Where we are headed

- In 2027 a Standard Galaxy (SG) will encapsulate our knowledge of the MW
- Like a Wiki page it will be a work in progress
- When a survey is planned, the SG will predict the survey's contents given its Selection Function (SF)
- When the survey is complete the SG will be updated by maximising the likelihood of the new data with respect to the SG's parameters and priors from earlier surveys
- The SG will describe what's out there, surveys will see just part of it

# What's in the SG?

- The distribution in  $(x,v)$  of many types of stars
  - K giants, BHB stars, RC stars, A stars, FG stars, K & M dwarfs, white dwarfs
  - Most types subdivided by age,  $[Fe/H]$  and  $[\alpha/Fe]$  and possibly other abundance ratios
- The distribution in  $(x,v)$  of dark matter
- Density of  $H_1$ ,  $H_2$ , perhaps density of  $H^+$

# What lies behind the SG?

- We can't avoid DM but we can't (yet?) directly see it
- We have to infer its distribution from the impact its contribution to  $\Phi(x,t)$  has on the distribution in  $(x,v)$  of stars & gas
- The distribution of tracers constrains  $\Phi$  only to the extent that the MW is in statistical equilibrium
- So we have to start from equilibrium models
- We can add non-equilibrium features (spiral arms, warp,..) later
- The natural way to specify an equilibrium is to use Jeans' thm to argue that *each component* has  $f(\text{integrals of motion})$ 
  - This result reduces each component to a distribution of stars in a 3d orbit space – easier to imagine than 6d phase space!

# AA variables

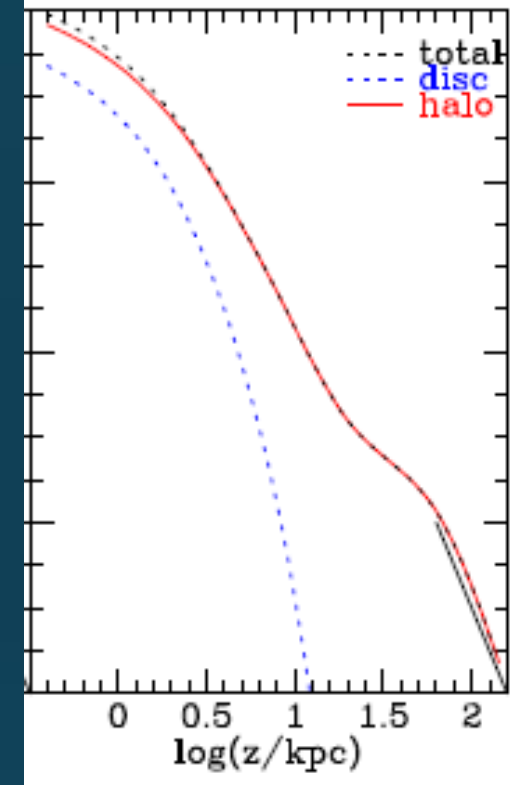
- Since any function  $g$  (integrals of motion) is an integral of motion, there are infinitely many possible choices for the arguments of  $f$ 
  - But there's only one sensible choice
  - To use actions  $J$  as our integrals
- Action are special
  - They are the only integrals that can be complemented by canonically conjugate coordinates (angle variables) to make up a complete set of coordinates
  - They are adiabatically invariant, so slow changes in  $\Phi$  don't cause any change in  $f$ 
    - makes it easy to compute response to secular change
  - They range from  $(0, \infty)$  or  $(-\infty, \infty)$  and  $(2\pi)^3 d^3J = d^3x d^3v$ 
    - This fact makes it possible to specify the mass of a component before solving for  $\Phi$
  - Their values have immediate physical meaning: In the simplest case:
    - $J_r$  quantifies radial oscillations,  $J_z$  quantifies oscillations perp to plane,  $J_\phi = L_z$  is angular momentum around symmetry axis
  - They are designed for use with *perturbation theory*

# Basic procedure (Binney 2014, Piffl+ 2015)

- Choose  $f(J)$  for each component (stars of given age & chemistry; DM)
- Guess  $\Phi(x)$  use  $J(x,v)$  (e.g. from Staeckel Fudge) to determine  $\rho(x)$  by integrating over  $v$  on grid in  $x$
- Solve Poisson eq for corresponding  $\Phi(x)$  and return to previous step
- Converges in 4-5 iterations
- If you allow  $E$  to enter  $f$  it's hard to achieve convergence, & if you succeed, convergence will be slow

# What's been done so far

- Much work with  $f(J)$  for stars but  $\rho(x)$  given for DM
  - Binney 2010, 2012, Bovy & Rix 2013, Piffl+ 2014, Trick+ 2016
- Then it's ok to assume parametrised  $\Phi(x)$  rather than getting  $\Phi(x)$  from Poisson's eq because reasonable to assume that
  - $\Phi_{\text{DM}}(x) = \Phi(x) - \Phi_*(x)$
- Less challenging computationally because you don't have to do 3d integral at points of  $(R, z)$  grid
- Downsides:
  - Can't predict kinematics of DM (direct detection experiments)
  - We expect  $f(J)$  of DM to be smooth but  $\rho(x)$  to have complex shape due to squeezing by disc (Binney & Wong 2017)
- Cole (last talk) will update on state of the art:
  - Axisymmetric disc & dark halo in self-consistent  $\Phi$
  - Strong constraints on  $\rho_{\text{DM}}(x)$  at  $R < R_0$

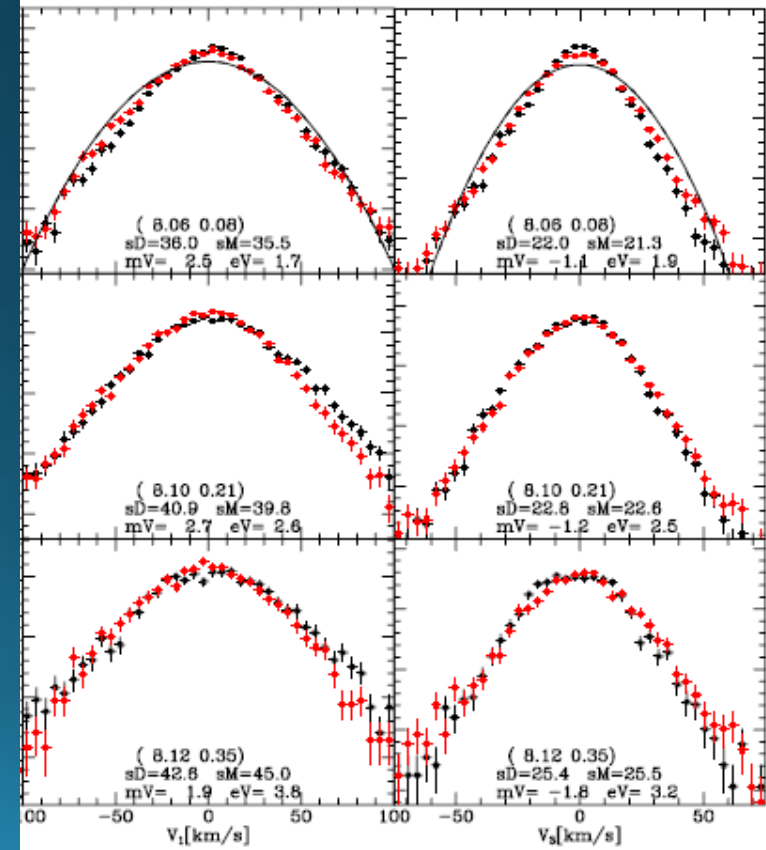
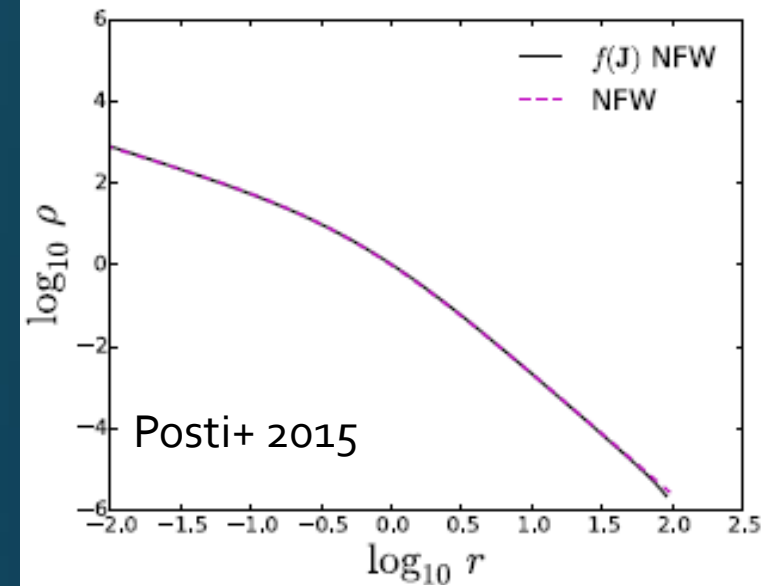


Binney & Wong 2017



# On choice of $f(J)$

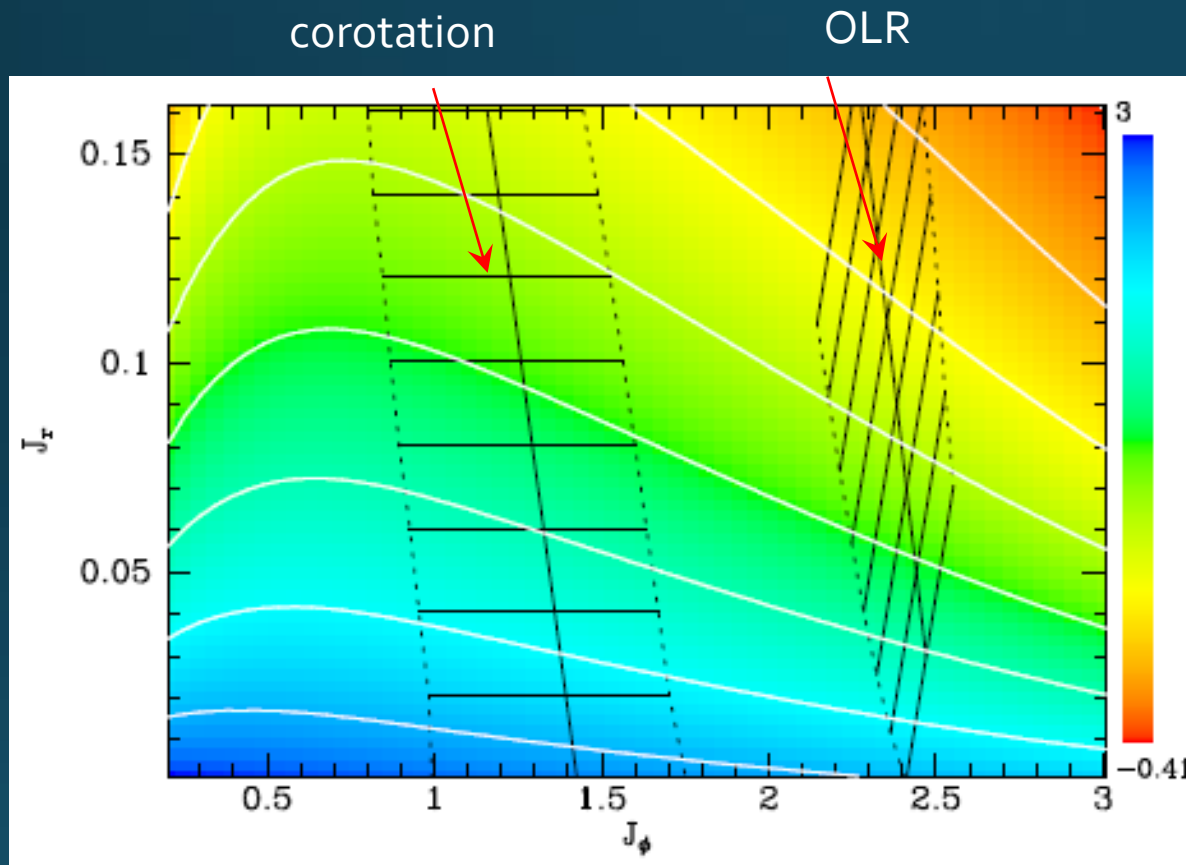
- Simple analytic functions generate excellent approximations to familiar models (Hernquist, NFW,..)
  - Posti+ 2015, Williams+ 2015
  - Models can be flattened, set rotating, whatever you want
- Models of the Galactic disc have been assembled from *quasi-isothermal* DF (Binney & McMillan 2011)
  - Fit data to extraordinary precision
  - Have demonstrated predictive power (Binney+ 2014)
- Quasi-isothermal DF was used by Bovy & Rix 2013 to model mono-abundance populations, but it cannot fit many such populations
  - We need to generalise
  - Sander & Binney (2015) suggest a direction of travel, but we should explore further



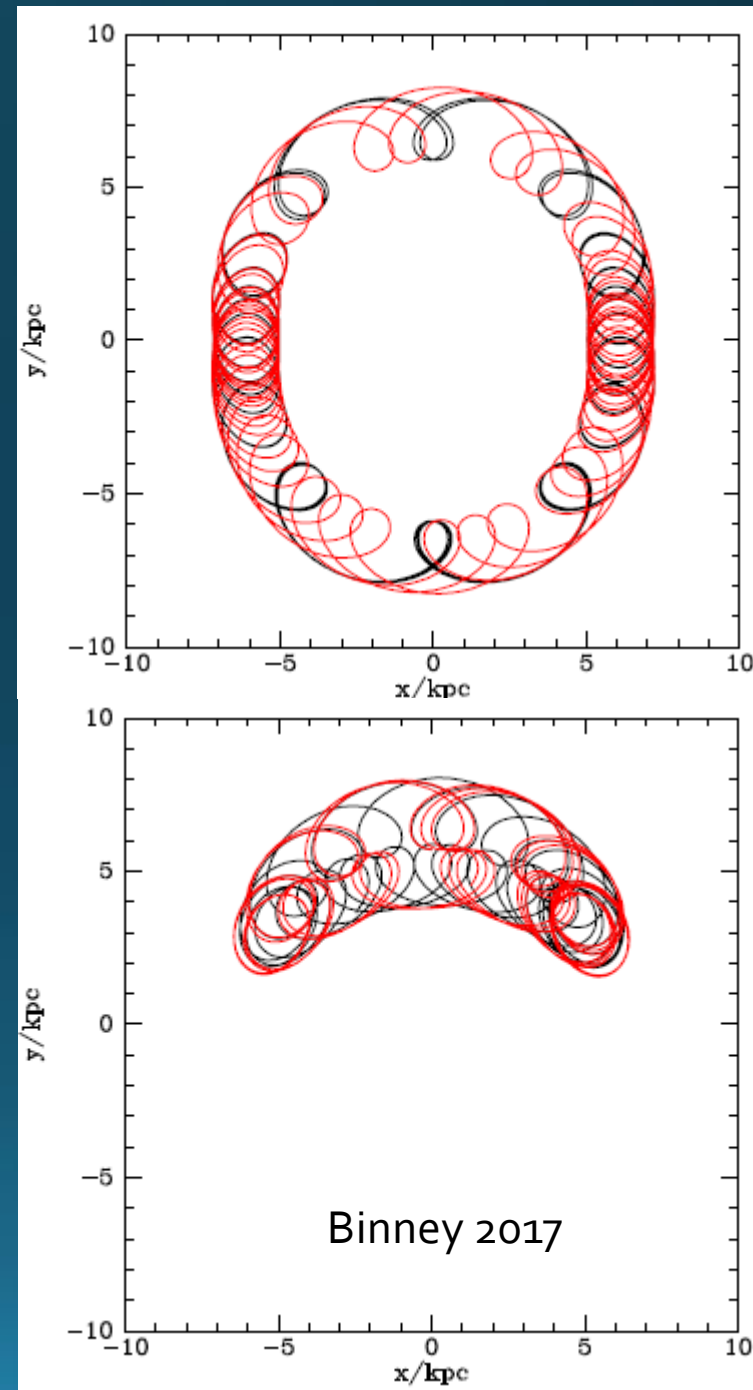
# Moving on

- Non-axisymmetry is of fundamental importance for MW
  - Aumer+ 2016a,b, 2017a,b
- The bar seems to extend to  $R \sim 6$  kpc & strongly affects  $S_{\text{nhd}}$ 
  - Perez-Villegas+ 2017, Portail+ 2017
- Spiral structure is  $\delta f(x,v,t)$  and should be modelled dynamically
  - Traditionally traced through  $\rho(x)$  but it has  $v$ -signature too
- $\Phi(x,t)$  can affect DFs in 2 ways
  - Non-resonant perturbation:  $J \rightarrow J + \delta J, \theta \rightarrow \theta + \delta\theta$
  - Resonant trapping: in restricted region of  $(x,v)$  old AA variables must be traded in for new ones
- Big problem: in trapped regions Staeckel Fudge doesn't work
- We are driven back to an orbit model: given  $(J,\theta)$  we can find  $(x,v)$  but not vice versa

# Impact of the bar



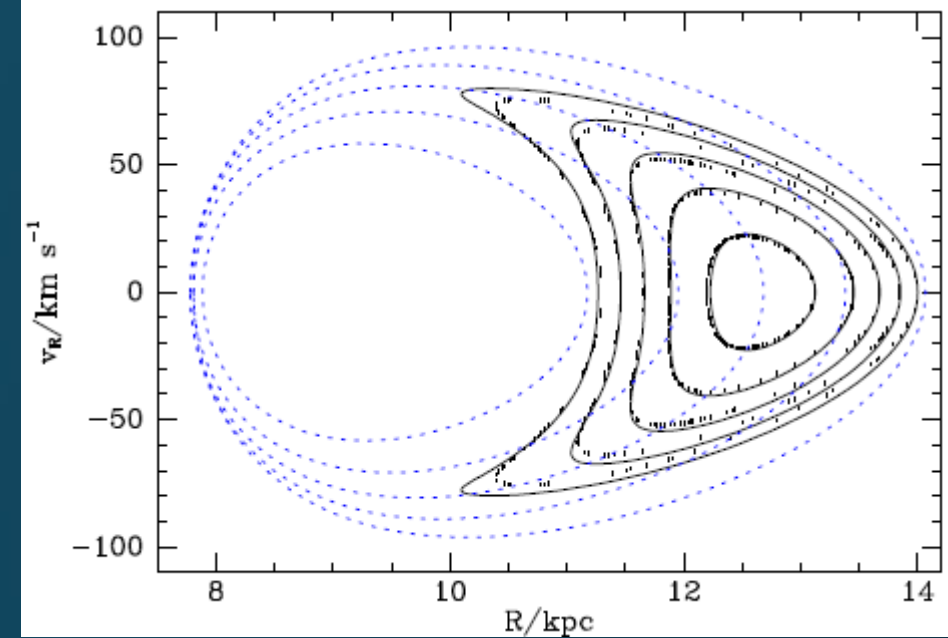
Colour give value of DF



# Traditional orbit models

- Schwarzschild models (Schwarzschild 1979, Cappellari+ 200, van der Ven 200)
  - Currently provides best models of external ETGs
- M2M models (Syer & Tremaine 1996, De Lorenzi+2007)
  - Currently provides the best models of the bulge/bar (Portail+ 2016)
- In these models
  - Initial conditions for orbit integration play role of integrals of motion
  - Weights of the orbits play role of  $f(\text{orbit})$
- It's relatively straightforward to fit models to data
- Non-axisymmetry & resonant trapping not problematic
- Major differences with  $f(J)$  modelling
  - Very many more parameters in DF and parameters lack evident physical meaning
  - Labels of orbits are complex and lack physical meaning
- I don't like these models because:
  - They are cumbersome: specified by millions of weights with low individual information content
    - Consequently
      - it's hard to compare models
      - It's hard to add components (DM, stars of various ages, chemistries,..)
  - DM is problematic because not directly constrained by data

# So back to $f(J)$ !



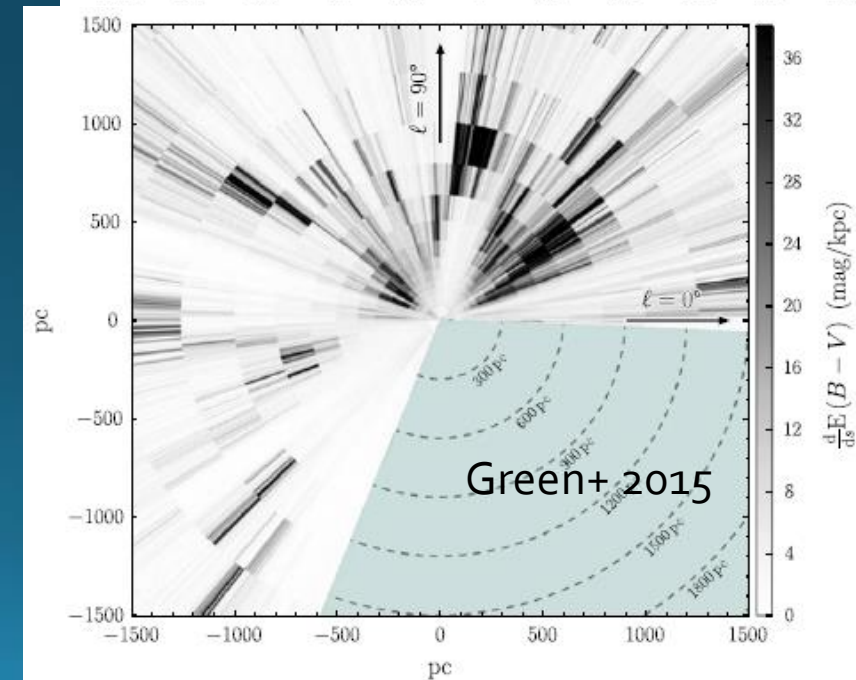
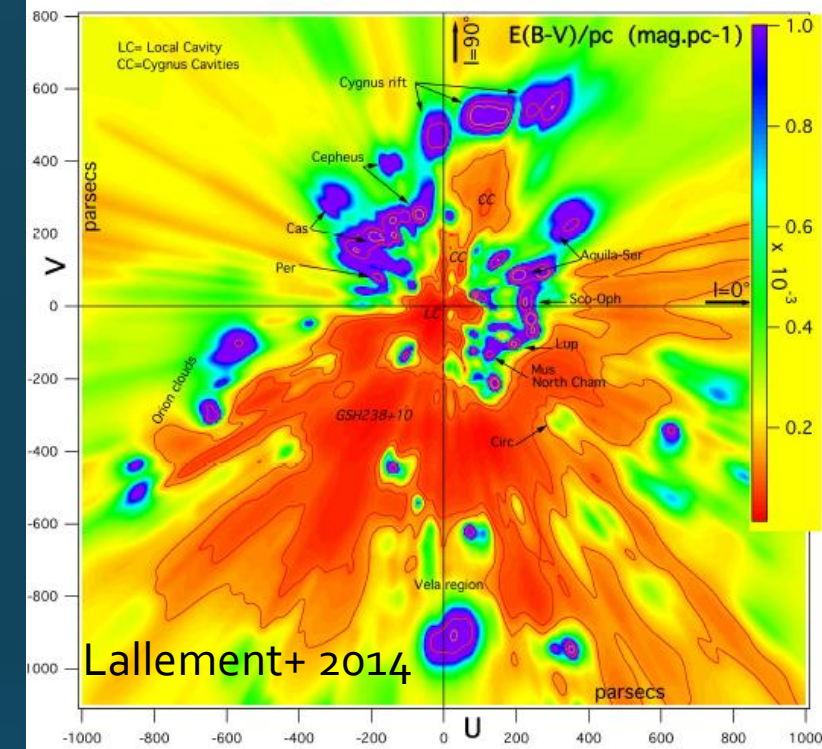
- For each component we need  $f(J)$  for non-trapped orbits
- For each major family of trapped orbits we need a new  $f(J)$
- To evaluate observables we must currently use  $x(J, \theta)$   $v(J, \theta)$  provided by Torus Mapper plus perturbation theory
- The good news is that perturbation theory works remarkably well
- It is, however, harder to fit  $(f, \Phi)$  to data when using  $x(J, \theta)$  than when using  $J(x, v)$  (McMillan & Binney 2013)
- Gaia data will help us determine  $f(J)$  for trapped orbits – no current knowledge of this

# Selection functions

- We need SF to predict contents of survey from a model
  - and hence by evaluating likelihoods to update the model
- The observability of a star depends on  $L$  in various bands, so on  $(\text{age}, [\text{Fe}/\text{H}])$ 
  - Hence DFs must depend on  $(\text{age}, [\text{Fe}/\text{H}])$
  - That is *history* is required to specify *current state*
- Our job now is to pin down current state not guess how MW was assembled
  - but a model history is needed to determine even current state
  - Model history does not need to be true so long as it gives valid  $f(J, \text{age}, [\text{Fe}/\text{H}])$
- Survey SF should be  $P(l, b, m \text{ in bands})$ 
  - stellar models convert to  $P(s, \text{age}, [\text{Fe}/\text{H}])$  which is handy when modelling
- Given importance of  $(\text{age}, [\text{Fe}/\text{H}])$  spectroscopic surveys are essential
  - So we must work largely with intersection of Gaia and APOGEE/RAVE/LAMOST/...
  - Gaia-TGAS SF is complex and involves  $s, v, \dots$  so hard to work with

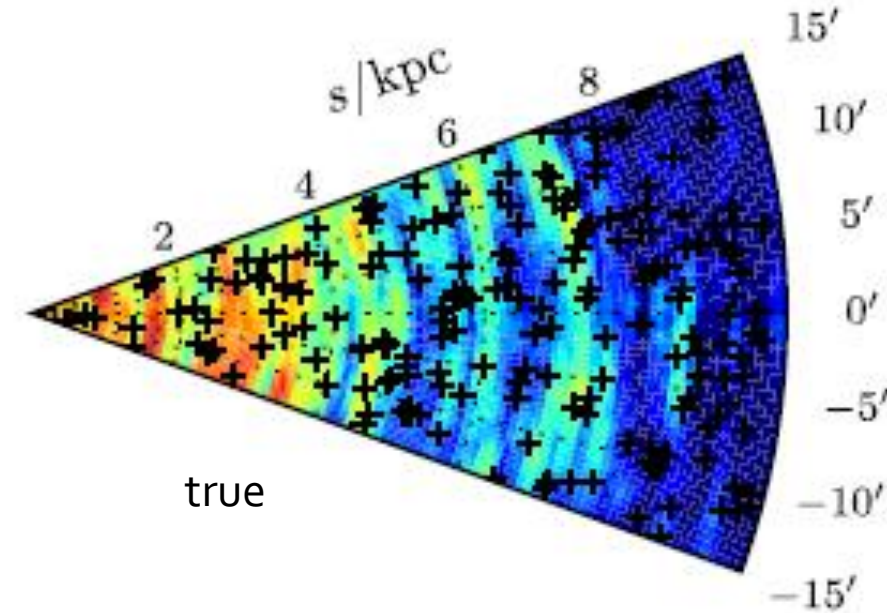
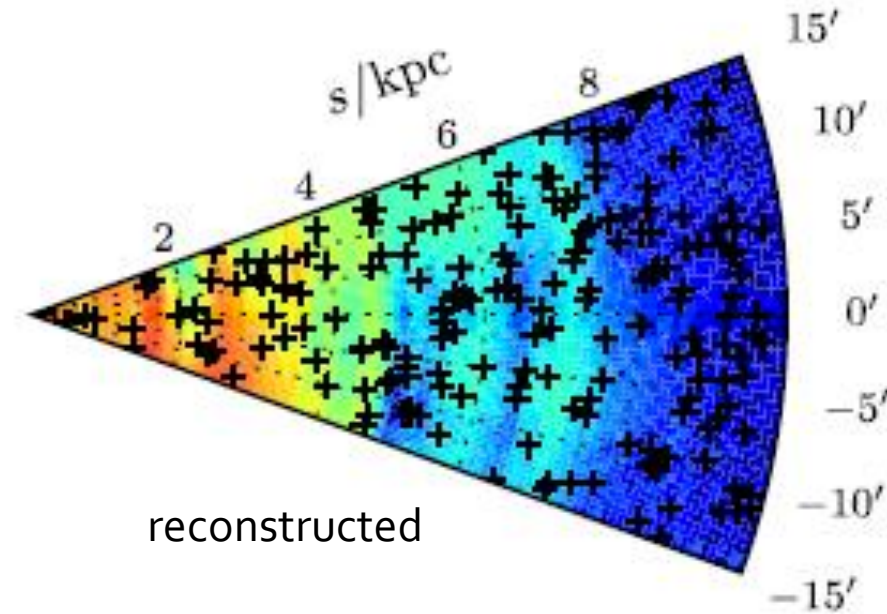
# The ISM

- At low  $b$  SF has significant dependence on ISM
- Hence we have to model ISM in parallel with stars
- Quality photometry for zillions of stars with trig parallaxes will be transformative
- Big issue is controlling Fingers of God



# Gaussian random field

- Must avoid modelling each line of sight in isolation
- Need to impose continuity on ISM also transverse to  $l$  or  $s$
- Simplest technique: represent ISM as Gaussian random field (Sale & Magorrian 2014)
- Computationally challenging





# Conclusions

- The SG is the ultimate fruit of our efforts
- It must be elaborated from an equilibrium axisymmetric model
- $f(J)$  modelling the most promising technology
- On account of SF we need  $f(J, \text{age}, [\text{Fe}/\text{H}])$  & we probably want other chemical coordinates
- So a model history is required even to specify current state
- P-theory provides a promising way of including the bar & spirals
- Sadly, for each family of trapped orbits we need a new  $f(J)$
- A model of the ISM has to be an integral part of the SG
- We have big computational challenges ahead